Field Precision LLC

Modeling a macroparticle electrostatic transport system with Trak

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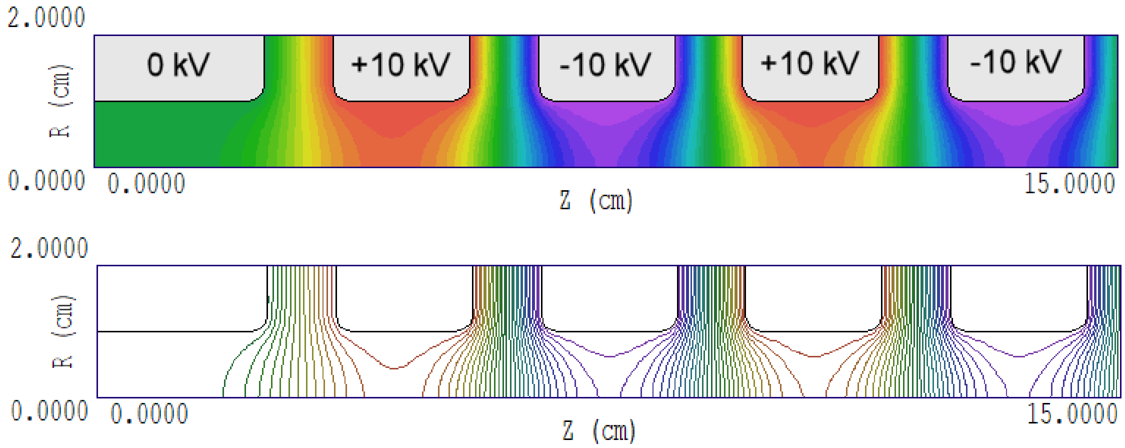

Figure 1: Section of an immersion lens array, normalized solution. The entrance electrode has potential 0.0 V and subsequent electrodes have potential ±1.0 V. Contour (upper) and contour line (lower) plots of electrostatic potential.

Although the **Trak** code is primary used to model beams of elementary particles like electrons and ions, the methods can be applied to particles of any mass and charge. This tutorial discusses electrostatic transport of *macroparticles*. We apply the term to particles that have much higher mass and charge than elementary particles, but are small enough to treat as point objects. In other words, the particle size is small compared to the dimensions of the transport system electrodes. A user suggested the following particle parameters: mass $M = 3.35 \times 10^{-11}$ kg, charge $Q = 1.0 \times 10^{-12}$ C and velocity $v = 40.0$ m/s. A requirement is to transport a beam of the particles with diameter greater than 1.0 cm a distance 100 cm in high vacuum. The goal of the calculation is to determine the maximum particle flux F set by space-charge effects.

We will consider the simplest transport system, the immersion lens array shown in Fig. 1. It consists of cylindrical rings with applied voltages of alternating polarity. The lower plot shows that the geometry creates radial electric field components of the form *Defocus/Focus* in a gap transitioning to a higher voltage (decerlating) and *Focus/Defocus* in a gap transitioning to a lower voltage (accelerating). On average trajectories have smaller radii in *Defocus* sections, so the net effect is focusing. In the system shown, the cylinders have length 2.0 cm and inner radius 1.0 cm. The gap width is 1.0 cm. The normalized voltages are multiplied by a scaling factor in the **Trak** runs. The full array length is 50.0 cm, sufficient to demonstrate beam matching. The rings at the entrance and exit are grounded.

There are two values required to define a **Trak** solution:

- The voltage multiplication factor.
- The matched particle beam current.

With regard to voltage multiplication, a useful quantity for comparison is the voltage necessary to accelerate the particles to velocity 40 m/s. The corresponding kinetic energy is $T = MV^2/2 =$

2.68×10^{-8} J. The acceleration voltage is $V_{ac} = T/Q = 26.8$ kV. The electrode voltage must be below this value but high enough to modify the particle trajectories significantly. We will use the multiplication factor 10^4 in the following calculations.

The beam current is determined by a balance between the focusing force of the lens array and the space-charge. We can estimate the average focusing force by performing a run in the *Track* mode with no space-charge force. If the quantity R_0 is the envelope radius, the radial force averaged over several cells in z is approximately linear and can be written as:

$$F(r) = F_0 \, \frac{r}{R_0}. \tag{1}$$

The equation of radial motion as a function of z is

$$\frac{d^2r}{dz^2} = -\left(\frac{F_0}{Mv^2}\right) \, \frac{r}{R_0}. \tag{2}$$

with solution

$$r = R_0 \, cos\left(\frac{2\pi z}{\lambda}\right). \tag{3}$$

Comparison of Eqs. 2 and 3 yields this expression for the axially-averaged radial force on the envelope:

$$F_0 = R_0 M v^2 \, \left(\frac{2\pi}{\lambda}\right)^2. \tag{4}$$

The envelope electric force on beam carrying current I is

$$F_e = \frac{QI}{2\pi\epsilon_0 v R_0}. \tag{5}$$

Setting Eqs. 4 and 5 equal gives the following expression for the matched current:

$$I = (2\pi)^3 \epsilon_0 \, v^3 R_0^2 \, \frac{M}{Q\lambda^2}. \tag{6}$$

Figure 2 shows a view of macroparticle trajectories in the absence of space-charge effects. Note that the radial scale is highly expanded to show details. The input distribution was generated with the *Circular beam generator* of **Trak**, discussed below. The spread in focal point positions is caused by small non-linearity of the focusing forces. Inspection of the figure shows that $\lambda \cong 0.5$ m for electrode voltages of ± 10.0 kV. Inserting all values in Eq. 6, we find that $I \approx 1.0$ μA, corresponding to a particle flux $F = I/Q = 10^5$ particles/second.

The *Circular beam generator* creates input particle distributions specifically for **Trak** in the form of `PRT` files. For this function, it is often better suited than the more general **GenDist** program. Figure 3 shows the parameters to create input for the main calculation. The beam radius value of 0.7 will be interpreted as 0.007 m in the **Trak** calculation with $DUnit = 100.0$. The file will contain 100 particles distributed uniformly in radius with weighted current assignment to give a uniform current density and total current of 1.0 μA. The rest mass value in atomic mass units is

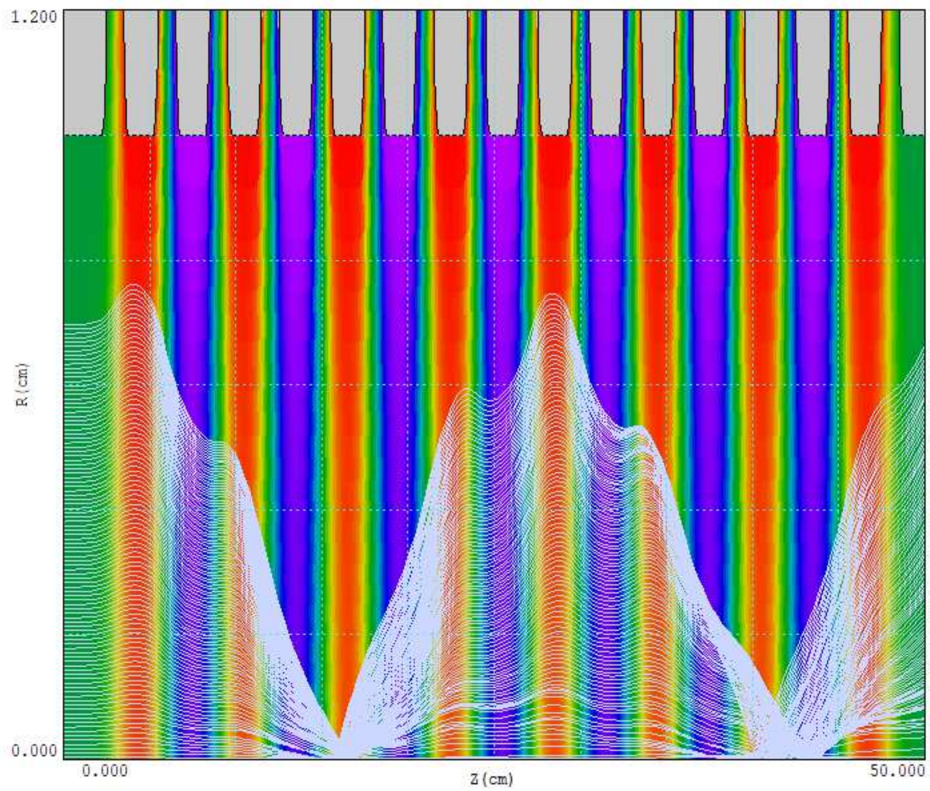

Figure 2: Radially-expanded view of macroparticle trajectories in the absence of space-charge forces.

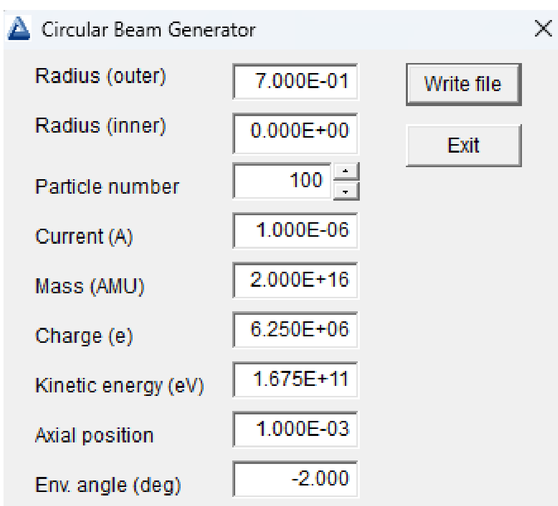

Figure 3: Setup in the *Circular beam generator* dialog with parameters for the production run.

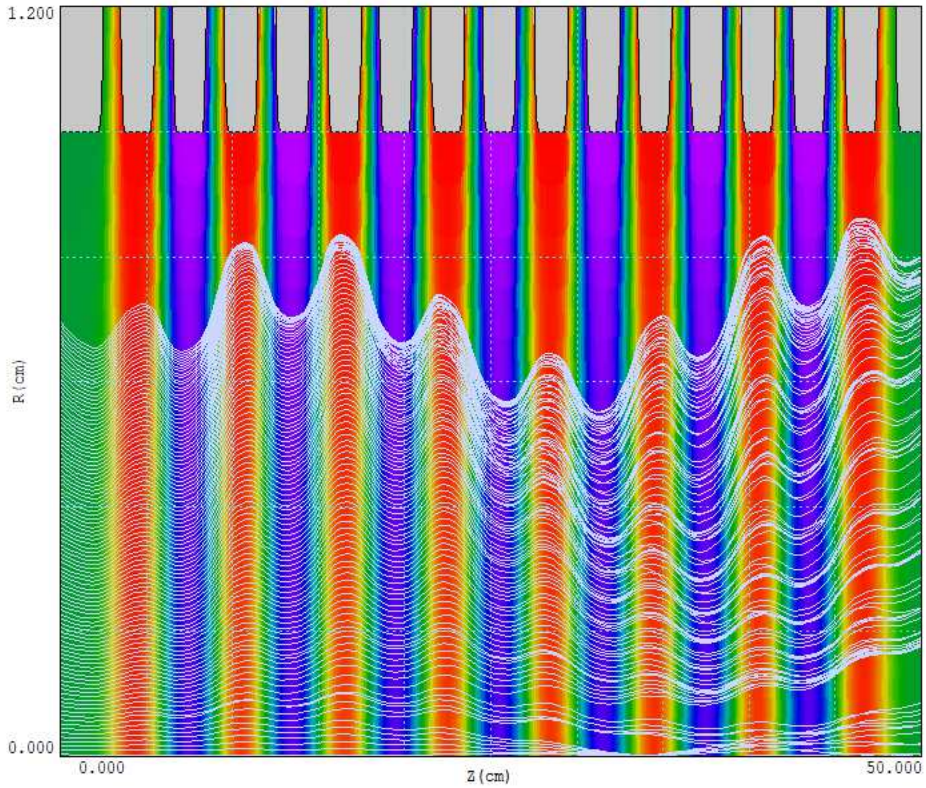

Figure 4: Radially expanded view of macroparticle trajectories for a 1.0 μA particle beam with self-consistent space-charge forces.

$$\frac{3.35 \times 10^{-11} \text{ kg}}{1.67 \times 10^{-27} \text{ kg/amu}} = 2.00 \times 10^{16} \text{ amu}. \tag{7}$$

The charge value in units of electron charge q_e is

$$\frac{1.00 \times 10^{-12} \text{ C}}{1.60 \times 10^{-19} \text{ C/q}_\text{e}} = 6.25 \times 10^{6} \text{ q}_\text{e}. \tag{8}$$

Dividing the kinetic energy T by 1.6×10^{-19} gives the input value 1.675×10^{11} eV. The axial injection position is just inside the entrance boundary. Finally, a small convergence with envelope angle -2.0$^\text{o}$ improves the match. Figure 4 shows the corresponding beam profile. The full beam current reaches the output port. The disordering of model particles is the result of the small focusing force non-linearity. To illustrate the effectiveness of the focusing system, if the electrode voltages are removed by setting the field adjustment factor to 0.0, the initially converging beam expands under the space-charge force. The envelope strikes a ring at z = 11.0 cm and only 0.17% of the current reaches the output port.