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How not to do a finite-element calculation

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The current zeitgeist is that software will do the thinking so that we can sit back and relax. This dream has been a continuing feature in the development of technical software – the idea that powerful programs will handle everything and friendly interfaces will free the user from worries about the physics. In searching the Internet to augment our example library, I found the perfect counter-example: a published study with seven authors from three institutions using powerful finite-element software that reports beautifully illustrated but incorrect conclusions. The work has so many problems that I felt it would be a worthwhile guide for students working with numerical codes. After reviewing the paper, I'll show that with a preliminary human analysis the calculation can be done correctly in a short time with great generality.

The paper, published in the **Proceedings of the 2020 IEEE International Conference on High Voltage Engineering and Application**, is *Electric Field and Temperature Distribution of High Voltage Cables with the Addition of Particles based on COMSOL Simulation* by A.A. Bhatti, *et.el.*[1]. The calculations deal with a specific high-voltage cable geometry with a specific insulator, ethylene propylene-rubber. The authors state: *The main purpose of this paper is to build a specific relationship between the particles in the insulation of the cable and the distance of particles from the conductor at multiple electrical potentials.* Figure 1 shows the simulation geometry. Electrostatic fields are calculated in the full cross-section of the coaxial cable with a two-dimensional simulation. A variety of *particles* with different composition (*i.e.*, relative dielectric constant), size and position are included in the insulator. Figure 2 shows plots of |`E`| for a different sets of particles. Parts *a* and *b* show an assortment of mica particles, while *c* shows the effect of water particles of different sizes. I quote the authors' conclusions, which appear to state that impurities in high voltage insulators are helpful:

- *Based on the simulation results, it has been observed that the electric field of the particles around the conductor area decreased as compared with the original electric field of the cable insulation.*
- *Based on the simulation results, it has been observed that the electric field of the particles around the conductor area decreased as compared with the original electric field of the cable insulation.*
- *It can be observed that electric field with different size of water particles decreased compared with original electric field distribution without any particles.*

We can proceed to consider the problems with the calculation procedure and the results.

1. To most people, I expect that the term *particle* implies an approximately spherical object (*e.g.*, air bubbles or water droplets). In the two-dimension calculation of the paper, the circular outlines actually represent long cylinders parallel to the cable axis, unlikely shapes for intrusions. Therefore, any reported field magnitudes would be misleading.

[1]The paper is available at `https://xplorestaging.ieee.org/document/9280024` and may also be downloaded from ResearchGate.

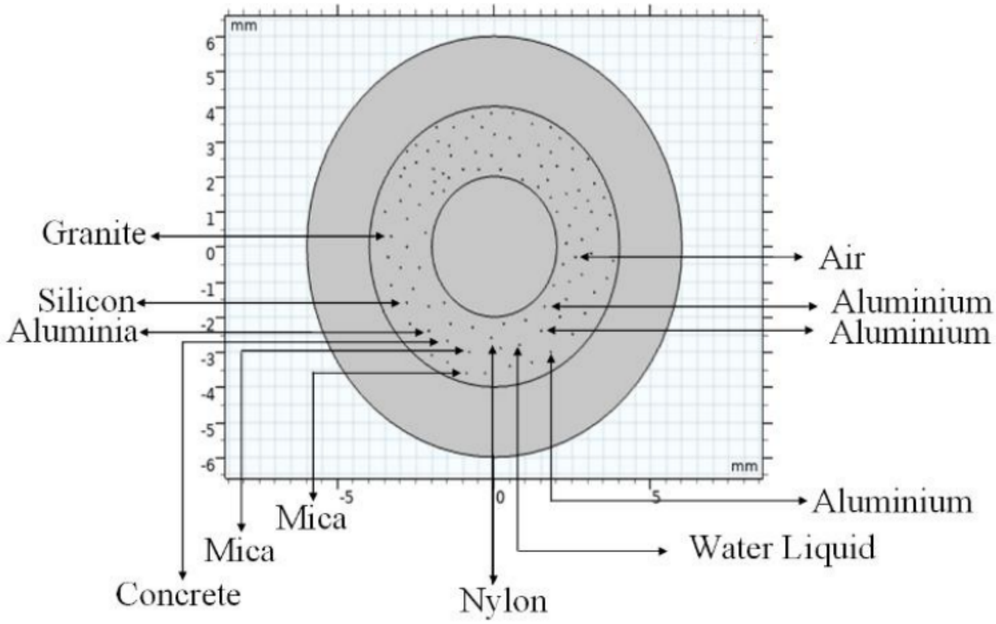

Figure 1: Geometry of the calculation reported in the A.A. Bhatti paper, showing a cross-section of the high-voltage cable in the two-dimensional calculation.

2. Modeling the full cross-section of the cable with tiny internal objects gives insufficient accuracy inside and near the particles. Realistic inclusions have small dimensions compared to scale lengths for variations of electric field. A local model of a dielectric particles in a uniform field would give much better accuracy.

3. The stated purpose and the motivation for the global model is to find the variation of the particle effect with position in the cable. We will see that the relative effect of small inclusions does not depend on field intensity, so there is no reason to move the particles around.

4. The calculations do not have a well-defined purpose or a criterion of success. The plots of Fig. 2a and b show little except that the mica particles appear to be surrounded by small yellow clouds.

5. Figure 2c for water rods shows an internal field comparable to or greater than the surrounding medium, clearly in error. The contention that water inclusions reduce the local field in a dielectric is incorrect.

6. By concentrating on a specific geometry and set of materials, the authors miss valuable general insights.

In summary, the faults of the paper result from 1) viewing the problem in a literal sense, 2) immediately turning to technical software as a black box and 3) accepting the code output without checking the physics. In the remainder of this tutorial, I will illustrate how to use finite-element

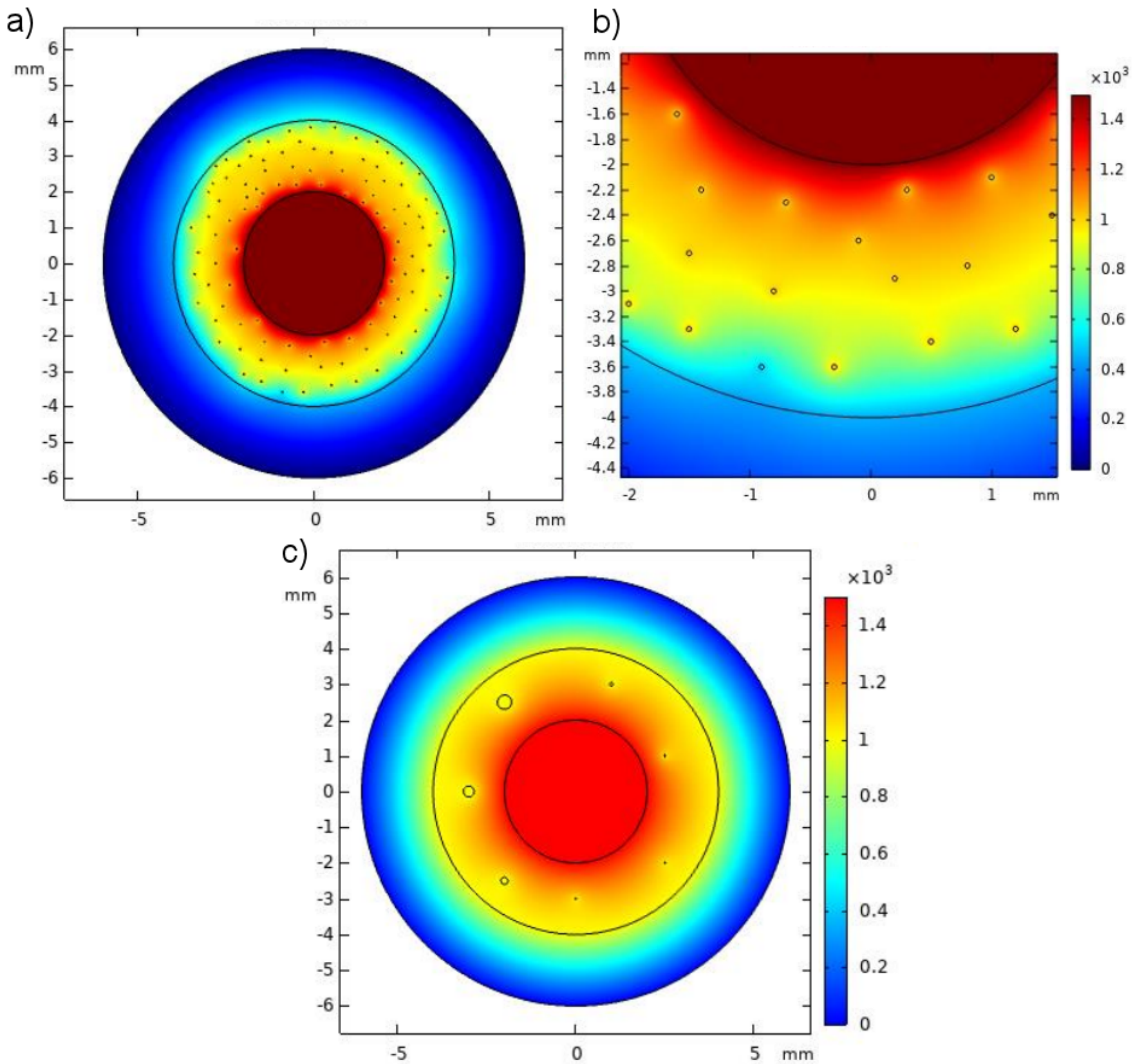

Figure 2: Results reported in the A.A. Bhatti paper. All plots show $|\mathbf{E}|$ as a function of position in the cable cross section. a) Inclusions of assorted mica rods b) Detail of the mica solution. c) Water rods of different sizes.

software to generate valid results. The first step is to state a clear purpose. Insulating dielectrics in high-voltage systems are often stressed near the breakdown point. Inclusions that create local field enhancements must be avoided. The goal of my calculation is to answer a simple question: how do spherical dielectric inclusions of different sizes and composition enhance local electric field values in high-voltage insulators?

As noted, if inclusions have small dimensions compared to the scale length of field variations, it is sufficient to build local models of small particles immersed in a uniform electric field. The advantage is that good accuracy can be achieved with reasonable number of elements. Fig. 3 shows the geometry of the calculation with dimensions in millimeters.[2] A spherical object with relative dielectric constant ϵ_b is immersed in a uniform medium with relative dielectric constant ϵ. The left and right boundaries have assigned potentials ± 5.0 mV. When $\epsilon_b = \epsilon$, the reference electric field is $E_o = 1.0$ V/m. Note the fine element resolution of the object for high accuracy. The function of the calculation is to answer two questions:

- How do fields in different parts of the solution space differ from unity when $\epsilon_b \neq \epsilon$?
- What are the locations and values of maximum field enhancement, $|\mathbf{E}|/E_o$?

Although the solution of Fig. 3 addresses a specific geometry and applied voltage, it can yield general results for any materials or geometry with the following insights:

- The electric field levels do not depend on the absolute values of ϵ_b and ϵ, but rather on the ratio ϵ_b/ϵ. A solution with $\epsilon_b = 10$ and $\epsilon = 1$ gives the same electric field distribution as one with $\epsilon_b = 25$ and $\epsilon = 2.5$.
- The size of the object does not affect the relative electric field distribution. For example, if we change dimensions to meters and set applied voltages ± 5.0 V (to maintain $E_o = 1.0$ V/m), the relative electric field distribution and magnitudes are unchanged.

Figure 4 shows a solution where $\epsilon_b/\epsilon \ll 1$ (such as an air bubble in water or transformer oil). The field inside the bubble is uniform with enhanced magnitude. Distortion of the equipotential lines results in reduced field magnitude at the top and bottom (relative to the electric field vector) surface of the sphere. The parallel component of electric field is continuous across a dielectric boundary so the value in the dielectric adjacent to the side of the object side equals the value in the interior. Figure 5 shows the opposite case where $\epsilon_b \gg \epsilon$ (such as a water droplet in oil). In this case, reduction of the field in the droplet pushes the equipotential lines outward, giving an enhanced field in the dielectric medium at the top and bottom of the sphere.

These solutions suggest performing a set of calculations varying ϵ_b with $\epsilon = 1.0$ and $E_0 = 1.0$ V/m and monitoring field values at the object center and just outside the object at the top. The calculations yield field enhancement $|\mathbf{E}|/E_0$ at the extreme points as a function of ϵ_b/ϵ for spheres of any radius and composition and electric fields of any amplitude. With scaling laws identified, specific numerical calculations can yield generalized results. The **EStat** input file for the sequence has the following content:

[2] Note that the calculation is performed with cylindrical coordinate weighting so that the half-circular object represents a sphere.

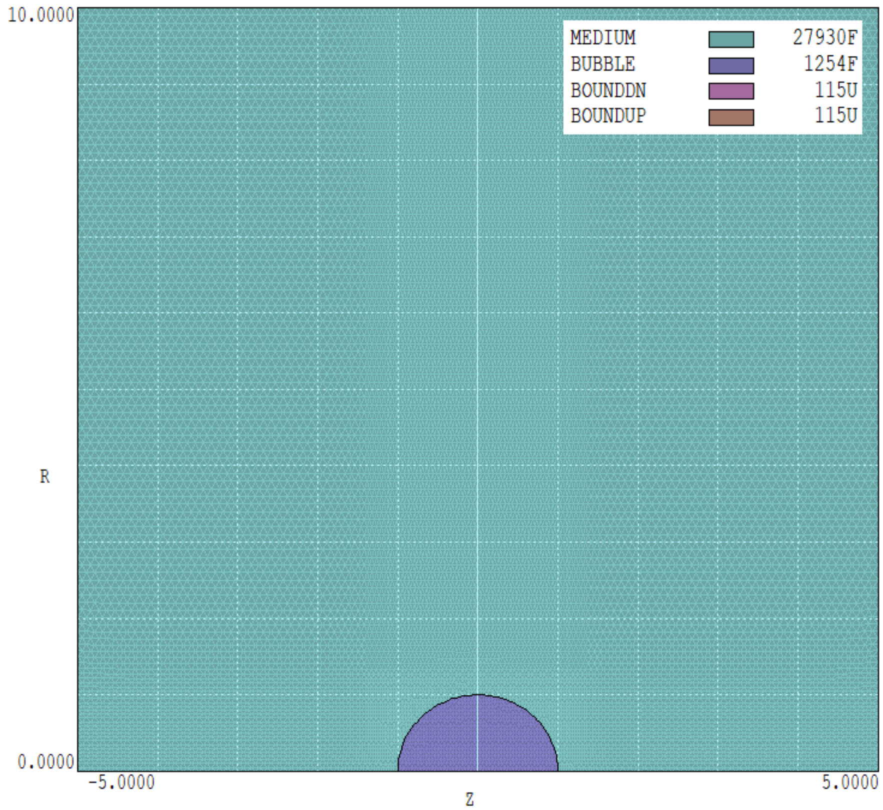

Figure 3: Geometry and mesh for finding local fields of a spherical dielectric object in a uniform electric field. Note that in the radial coordinates, the object is a sphere. Dimensions will be interpreted in millimeters.

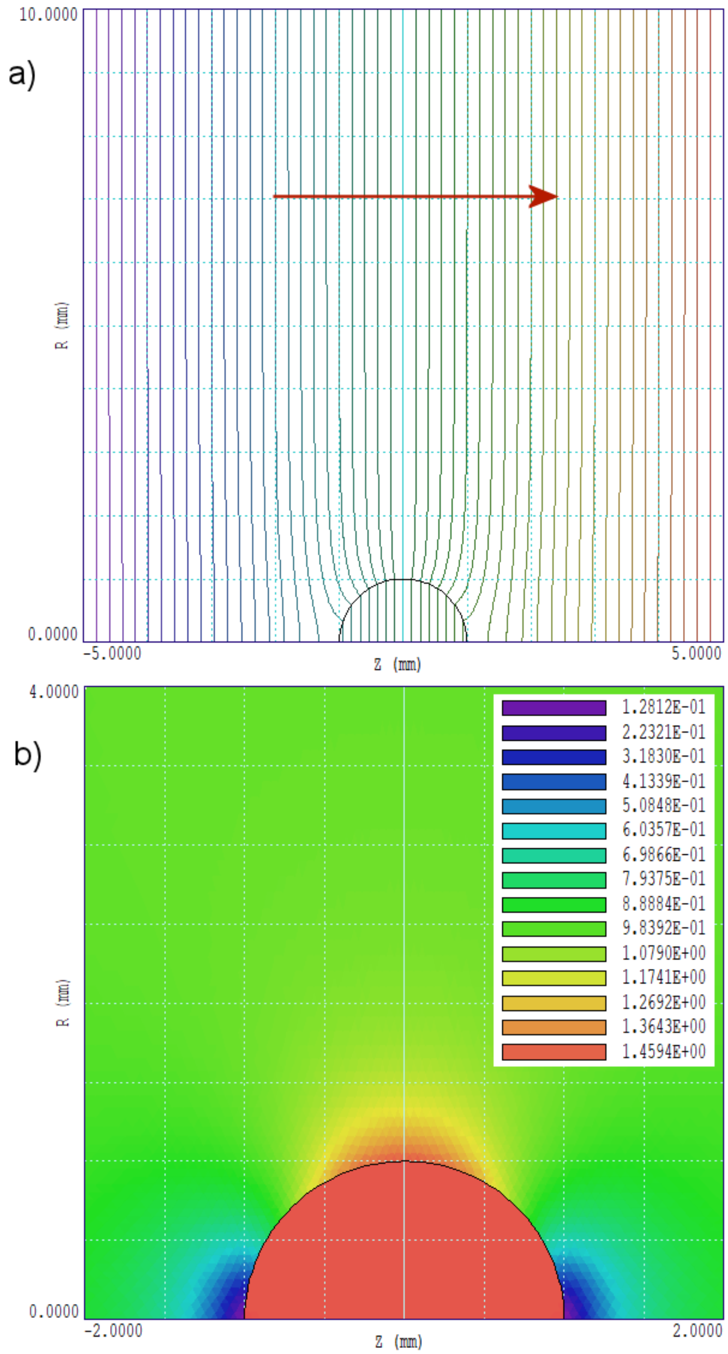

Figure 4: Solution with $\epsilon_b/\epsilon = 0.01$. *a*) Equipotential lines. The arrow shows the electric field direction. The top and bottom of the sphere (relative to the field) are on the left and right sides in the illustration. *b*) Magnified view of $|\mathbf{E}|$ inside and near the object.

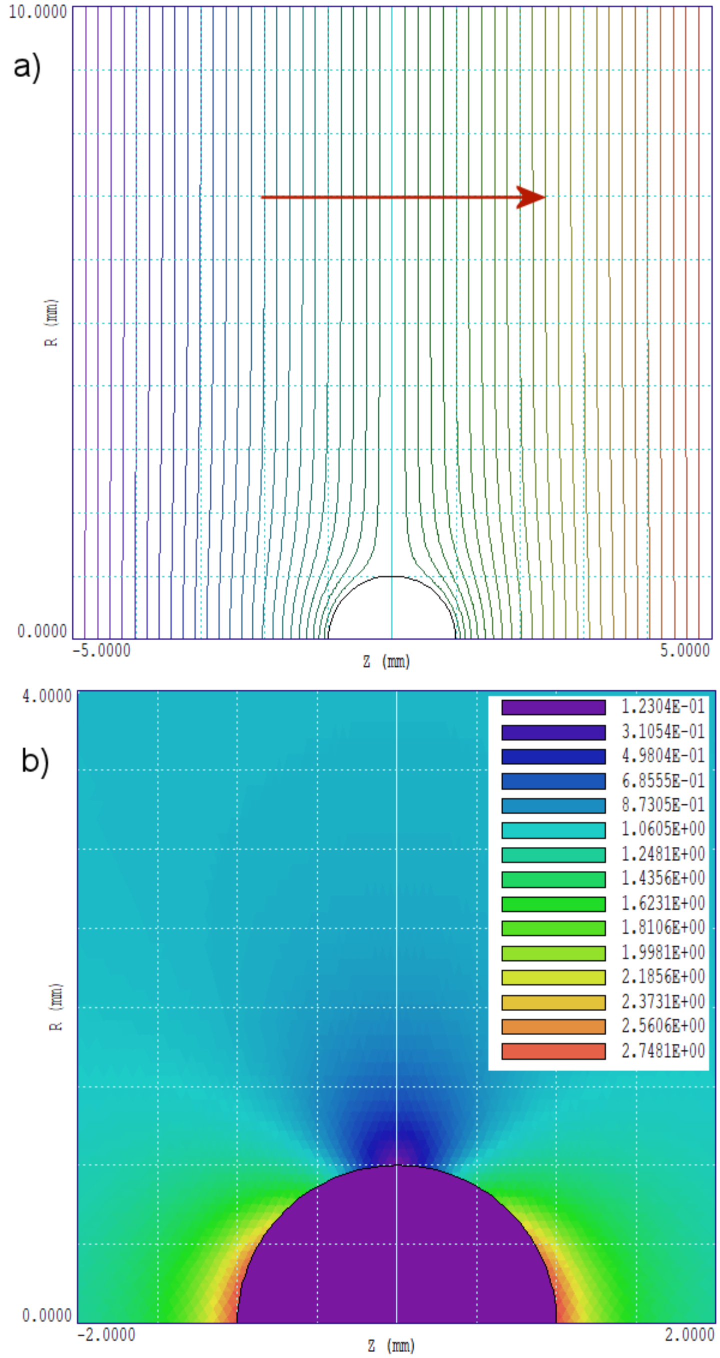

Figure 5: Solution with $\epsilon_b/\epsilon = 100.0$. *a*) Equipotential lines. *b*) Magnified view of $|\mathbf{E}|$ inside and near the object.

```
Mesh = Bubbles
Geometry = Cylin
DUnit =   1.0000E+03
ResTarget =   1.0000E-09
MaxCycle =    5000
* Region 1: MEDIUM
Epsi(1) =   1.0000E+00
* Region 2: BUBBLE
Epsi(2) = %1
* Region 3: BOUNDDN
Potential(3) =  -5.0000E-03
* Region 4: BOUNDUP
Potential(4) =   5.0000E-03
EndFile
```

Three of the commands set the dielectric constant of the medium to 1.0 and the electrode potentials to generate a uniform field $E_z = 1.0$ V/m in the absence of the bubble. In the region command for the bubble dielectric constant, the symbol %1 represents the value of a pass parameter when **EStat** is called from a batch file.

The calling file `Bubbles.BAT` has the content:

```
IF EXIST Abstract.txt ERASE Abstract.txt
ECHO EpsiB: 0.01 >> Abstract.txt
START /B /WAIT C:\fieldp/tricomp/estat.exe Bubbles.EIN 0.01
START /B /WAIT C:\fieldp/tricomp/estat.exe Bubbles.SCR
FINDSTR /L /C:"|E|:" Bubbles.DAT >> Abstract.txt

ECHO EpsiB: 0.02 >> Abstract.txt
START /B /WAIT C:\fieldp/tricomp/estat.exe Bubbles.EIN 0.02
START /B /WAIT C:\fieldp/tricomp/estat.exe Bubbles.SCR
FINDSTR /L /C:"|E|:" Bubbles.DAT >> Abstract.txt

...

ECHO EpsiB: 100.00 >> Abstract.txt
START /B /WAIT C:\fieldp/tricomp/estat.exe Bubbles.EIN 100.00
START /B /WAIT C:\fieldp/tricomp/estat.exe Bubbles.SCR
FINDSTR /L /C:"|E|:" Bubbles.DAT >> Abstract.txt
```

The commands specify the following actions:

- Delete any previous occurrence of the data file `Abstract.txt`.
- Write the current value of ϵ_b/ϵ to the data file.
- Run an **EStat** calculation with a specified value of ϵ_b/ϵ.
- Run an **EStat** analysis controlled by the script `Bubbles.SCR` to record values of $|E|$ at the extreme points.
- Transfer the values to the data file.

The sequence covers thirteen values of ϵ_b/ϵ in the range 0.01 to 100.0.

The analysis script `Bubbles.SCR` has the following content:

```
INPUT Bubbles.EOU
OUTPUT Bubbles.DAT
* Center inside
POINT 0.0,0.0
* Top outside
POINT 1.01,0.00
ENDFILE
```

It directs **EStat** to load the current solution file and to record a variety of quantities in an analysis file at specified points (z, r). The listing for a point calculation in the file `Bubbles.DAT` looks like this:

```
Field calculation at a point
ZPoint:  0.00000000E+00
RPoint:  0.00000000E+00
Phi:  1.14526656E-08
Ez: -2.95941640E-02
Er:  1.88757399E-07
|E|:  2.95941640E-02
Dz: -2.62032262E-11
Dr:  1.67129331E-16
|D|:  2.62032262E-11
EngDens:  3.87731287E-13
```

The `FINDSTR` batch command opens the analysis file and records all lines that contain the string `|E|:` in the data file `Abstract.txt`. At completion, the data file has the content:

```
EpsiB: 0.01
|E|:  1.48788739E+00
|E|:  5.90286319E-02
  ...
EpsiB: 100.00
|E|:  2.95941640E-02
|E|:  2.85094392E+00
```

The run time for the full set of calculations is less than 10 seconds. Values from `Abstract.txt` are ported to a plotting program to create Fig. 6, a universal curve for spherical dielectric objects. The left side of curve corresponds to case like an air bubble in water and the right side to a water droplet in air. The symbols represent numerically calculated values at the measurement points. For comparison, analytic theory[3] predicts that the electric field inside a sphere immersed in a uniform applied field is uniform with magnitude:

$$|\mathbf{E}|/E_0 = \frac{3}{\epsilon_b/\epsilon + 2}. \tag{1}$$

The solid line in Fig. 6 follows Eq. 1, confirming high numerical accuracy for the mesh of Fig. 3.

I made a calculation like that of Fig. 2c to show what the electric field should actually look like. As shown in Fig. 7, the space between coaxial electrodes is filled with transformer oil. The space contains cylinders of water of different diameters. The high dielectric constant material expels equipotential lines, giving field enhancement at the top and bottom of the rods relative to the electric field direction. The behavior of each rod is similar to the spheres of Figs. 5 As expected, the field amplitudes inside the rods and at the poles are almost equal, independent of the rod size.

My calculations lead to some conclusions about impurities in insulators. The electric field inside air bubbles in liquid or solid dielectrics can be as much as 50% higher than surrounding values. The lower breakdown level of air could cause ionization and degrade insulator stability. That is the reason we put debubblers in water-filled pulselines in the late bronze era. At the other extreme, water bubbles in liquid dielectrics and droplets in gas insulators can lead to local field enhancements by almost a factor of three, a significant concern. I emphasize that my calculations are limited to spherical inclusions. It is relatively easy to build models of alternate shapes with two or three-dimensional electrostatic programs. A final thought is a concern about making interfaces and operation of technical programs too friendly and too autonomous. Inexperienced users can generate impressive looking but flawed results that may wind up as publications.

[3]see, for instance, J.D. Jackson, Classical Electrodynamics, Second Edition (Wiley, New York, 1975), Sect. 4.4.

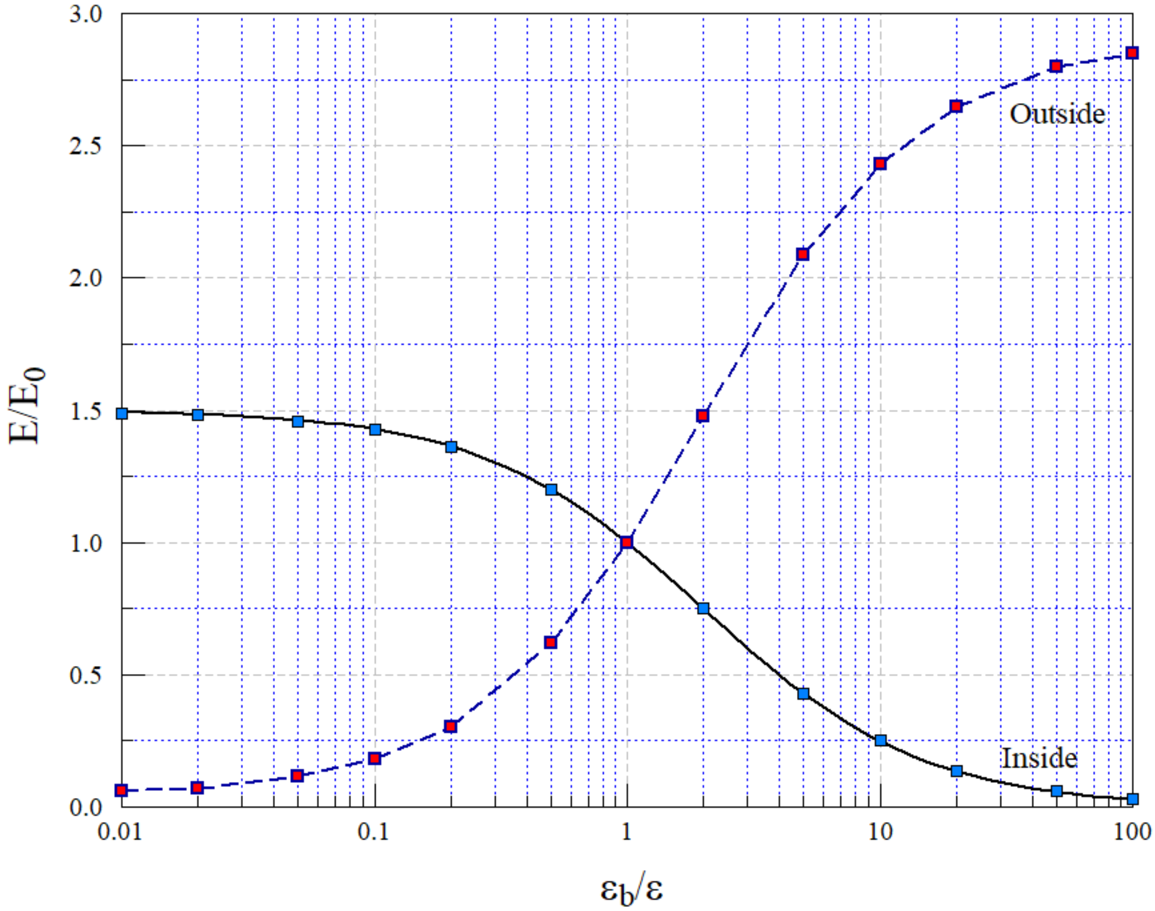

Figure 6: Electric field enhancement factors at the center (*Inside*) and top surface (*Outside*) of a sphere of any radius with relative dielectric constant ϵ_b in a medium with relative dielectric constant ϵ in a uniform applied field E_0.

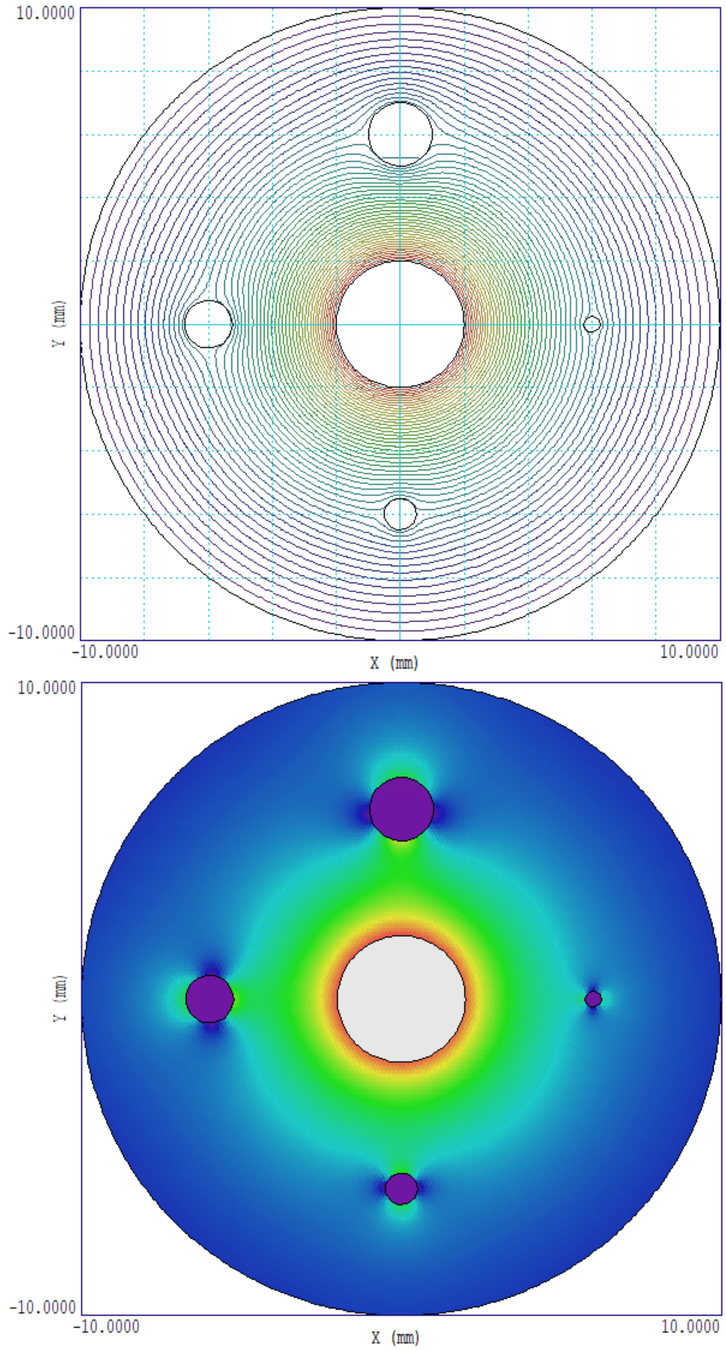

Figure 7: Rods of water ($\epsilon_r = 81.0$) of different radii in the space between coaxial electrodes filled with transformer oil ($\epsilon_r = 2.7$). a) Equipotential lines. b) Electrical field magnitude.