Modeling an electrostatic macroparticle beam deflector

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Figure 1: Twelve electrode system to generate an approximately uniform deflection field.

This tutorial is the second in a series addressing the following issue: implementation of a curved electrostatic transport system for charged particles. A practical system would include focusing and deflection field components within a cylindrical transport tube. A previous report, *Modeling a cylindrical charged-particle beam deflector*[1], described a method to approximate a uniform transverse electric field with multiple electrodes in a cylindrical volume of infinite length. This report extends the analysis to a finite length system, illustrating methods to determine edge effects and the uniformity of deflection. A second function is to show how to treat macroparticles in **GenDist** and **OmniTrak**. Although the programs are geared to handle elementary particles like electrons and ions, they can calculate trajectories for particles of any mass and charge. We use the term macroparticle to designate an object that is small compared to the system size but has mass and charge much greater than an elementary particle.

To begin, consider generation of the electrostatic field solution using **MetaMesh** and **HiPhi**. Figure 1 shows the electrode geometry, similar to that of the previous report. The propagation direction is along the z axis. The interior field has little dependence on the cross-section shape of the electrodes, so the electrodes are taken as cylinders rather than wedges for simplicity. Twelve cylinders of diameter 0.4 cm and length $L = 3.0$ cm are arrayed uniformly on a circle of radius 1.1 cm. The assembly is located in a grounded cubic box with sides of length 5.0 cm that could represent boundaries and adjacent electrodes of a transport system. The initial stages of mesh definition are carried out in **Geometer**. The solution volume (Region 1, Vacuum) is box with dimensions $-2.5 \leq x, y, z \leq 2.5$. A second region (Elect01) is a cylinder with radius 0.2 and length 3.0 with a displacement vector (1.10, 0.00, 0.00). At this point we could continue continue

[1] `https://www.fieldp.com/tutorials/CylindricalBeamDeflector.pdf`

in **Geometer**, duplicating and shifting electrodes to create the array. Alternatively, it is easy to perform the operations directly on the input script with a text editor. In this case, save the mesh file and open it in a text editor. (The tutorial *Integrating the NotePad++ editor with Field Precision programs*[2] describes how to add an editor with specialized syntax highlighting).

With a text editor, we can make several changes to the basic script. For example, the foundation mesh specification can be expanded with variable resolution to achieve good accuracy with a short run time:

```
XMesh
   -2.50000E+00 -1.50000E+00  5.00000E-02
   -1.50000E+00  1.50000E+00  2.50000E-02
    1.50000E+00  2.50000E+00  5.00000E-02
End
YMesh
   -2.50000E+00 -1.50000E+00  5.00000E-02
   -1.50000E+00  1.50000E+00  2.50000E-02
    1.50000E+00  2.50000E+00  5.00000E-02
End
ZMesh
   -2.50000E+00  2.50000E+00  5.00000E-02
```

The set of region names can be easily extended using copy/paste operations and changing a few numeric values:

```
RegName(  1): Vacuum
RegName(  2): Elect01
RegName(  3): Elect02
RegName(  4): Elect03
RegName(  5): Elect04
RegName(  6): Elect05
RegName(  7): Elect06
RegName(  8): Elect07
RegName(  9): Elect08
RegName( 10): Elect09
RegName( 11): Elect10
RegName( 12): Elect11
RegName( 13): Elect12
RegName( 14): Boundary
```

Finally, the initial electrode centered at (1.10,0.00,0.00) can be used as a template for the other electrodes by 1) copying the `Part` section and pasting it 11 times and 2) changing the region designation and the position in the x-y plane to reflect 30^o rotations:

[2]https://www.fieldp.com/tutorials/TextEditor.pdf

```
PART
  Region: Elect01
  Name: Elect01
  Type: Cylinder
  Fab:   2.00000E-01  3.00000E+00
  Shift:   1.10000E+00  0.00000E+00  0.00000E+00
END
PART
  Region: Elect02
  Name: Elect02
  Type: Cylinder
  Fab:   2.00000E-01  3.00000E+00
  Shift:   9.52600E-01  5.50000E-01  0.00000E+00
END
...
```

Figure 1 shows the end result. Note that conformal fitting is not applied because small variations in the surface shape have negligible effect on the internal fields.

The HiPhi input script `EDeflect3D.HIN` has the content:

```
Mesh = EDeflect3D01
DUnit =   1.0000E+02
ResTarget =   5.0000E-08
MaxCycle =    2500
* Region 1: VACUUM
Epsi(1) =   1.0000E+00
* Region 2: ELECT01
Potential(2) =  -1.0000E+00
* Region 3: ELECT02
Potential(3) =  -8.6600E-01

...
* Region 13: ELECT12
Potential(13) =  -8.6600E-01
* Region 14: BOUNDARY
Potential(14) =  0.0000E+00
EndFile
```

The voltages follow the equation discussed in the previous tutorial,

$$\phi_n = V_0 \ \cos[2\pi(n-1)/12], \tag{1}$$

where n is the electrode number in the range 1 through 12. Figure 2 shows the field solution in the plane $z = 0.0$ cm. The top plot of equipotential lines confirms that the electric field in the center portion points almost entirely in the x direction. The lower contour plot of E_x shows the

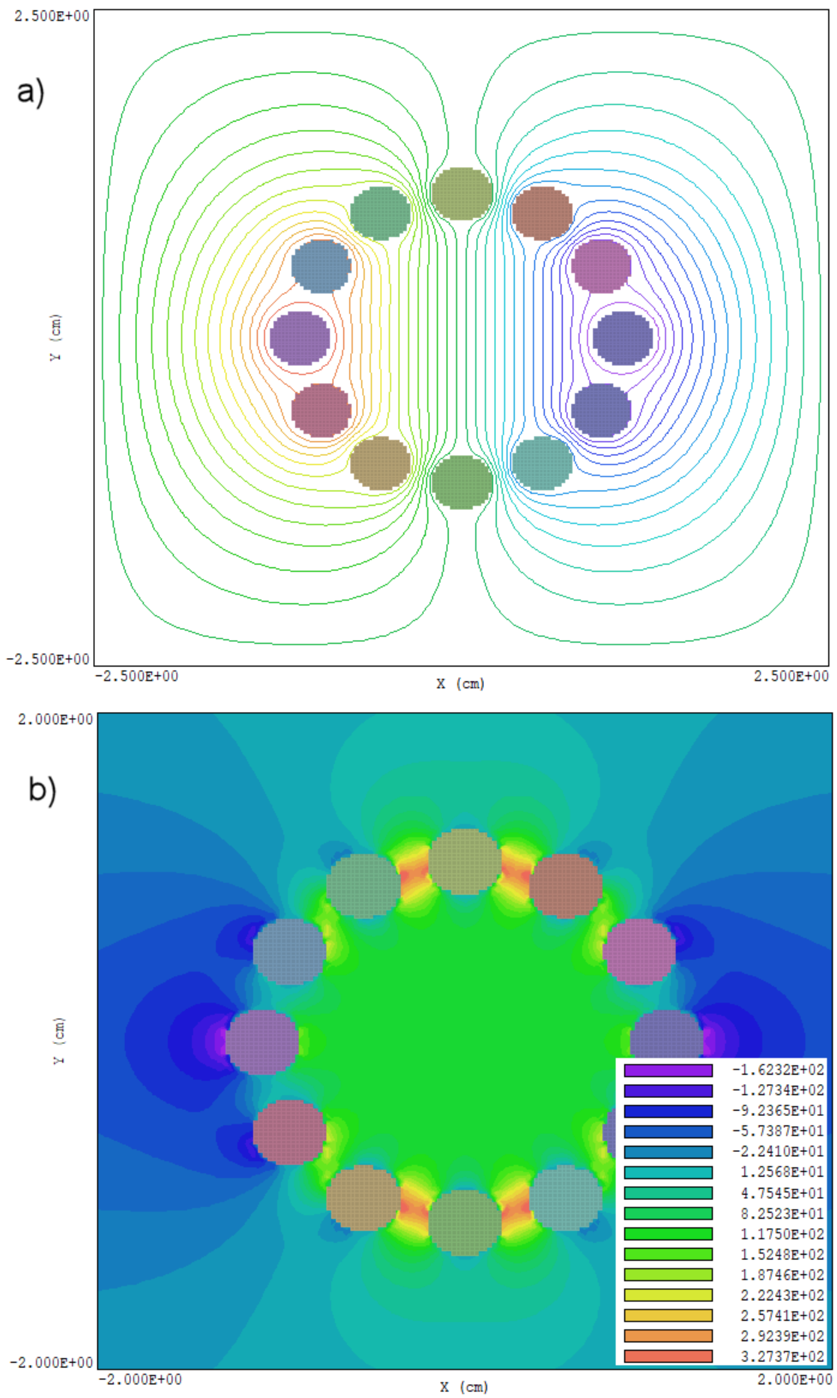

Figure 2: Electric field solution in the plane $z = 0.0$ cm. a) Equipotential lines. b) Contour plot of E_x.

uniformity of the interior deflection field magnitude. The transverse field value in the interior region is $E_x \cong 106.3$ V/m.

We next move the **OmniTrak** solution. An initially parallel circular beam starting at $z = -2.5$ cm moves through the deflector. We determine the beam shape and the distribution of transverse angles at the output ($z = 2.5$ cm). To demonstrate a setup for macroparticles, we will use the following parameters for spherical beam particles suggested by a user: mass $M = 3.35 \times 10^{-11}$ kg, charge $Q = 1.0 \times 10^{-12}$ C and velocity $V = 40.0$ m/s. The mass implies a particle radius of about 0.002 cm, so it is sufficient to treat objects as point particles. The **GenDist** input file `EDeflect3D.DST` to create about 500 particles in a beam parallel to z uniformly distribution over a circle of radius 0.7 cm at $z = -2.499$ cm has the content:

```
FileType = PRT
RestMass =   2.0000E+16
Charge =   6.2500E+06
Energy =   1.6750E+11
Def(UniCirc) =  7.0000E-01  500
Shift =   0.0000E+00  0.0000E+00 -2.4990E+00
Distribution = Uniform
```

The following relationships determine the particle parameters. The rest mass value in amu is

$$\frac{3.35 \times 10^{-11}\ \text{kg}}{1.67 \times 10^{-27}\ \text{kg/amu}} = 2.00 \times 10^{16}\ \text{amu}. \tag{2}$$

The charge value in units of electron charge q_e is

$$\frac{1.00 \times 10^{-12}\ \text{C}}{1.60 \times 10^{-19}\ \text{C/q}_\text{e}} = 6.25 \times 10^{6}\ \text{q}_\text{e}. \tag{3}$$

The kinetic energy in joules is

$$\frac{1}{2}\ (3.35 \times 10^{-11}\ \text{kg})\ (40.0\ \text{m/s})^2 = 2.68 \times 10^{-8}\ \text{J}. \tag{4}$$

Dividing the energy by 1.6×10^{-19} gives the input value 1.675×10^{11} eV. A useful quantity is the voltage V_{ac} necessary to accelerate the object to the kinetic energy of Eq. 4:

$$V_{ac} = \frac{2.68 \times 10^{-8}\ \text{J}}{1.0 \times 10^{-12}\ \text{C}} = 2.68 \times 10^{4}\ \text{V} = 26.8\ \text{kV}. \tag{5}$$

Accordingly, we expect that the applied deflector voltage should be in the kV range,

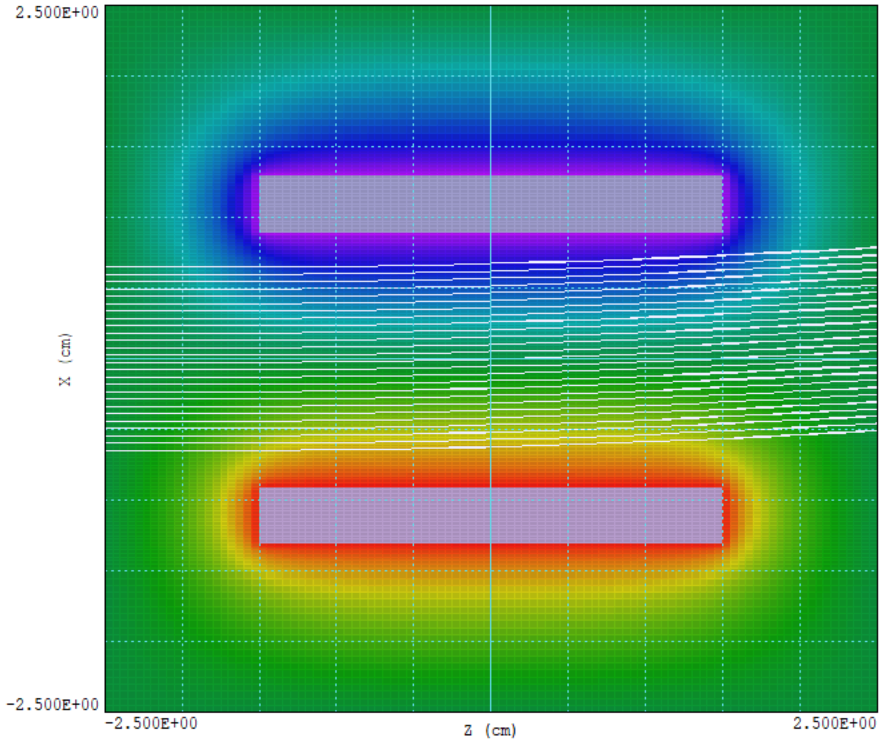

Figure 3: Projected particle orbits and potential contours in the plane $y = 0.0$ cm.

Input to **OmniTrak** is prepared in the `Set up script` dialog. The script generates the file `EDeflect3D.OIN` with the content:

```
FIELDS
  EFIELD3D: EDeflect3D.HOU 750.0
  DUNIT:   1.0000E+02
END
PARTICLES TRACK
  PFILE: EDeflector3D
  DT:   1.2000E-05
END
DIAGNOSTICS
  PARTFILE: EDeflector3DP
END
ENDFILE
```

The multiplication factor in the command to load the electric field corresponds to applied electrode potential limits of ±750 V with a central field of $E_x = 79.7$ kV/m. Figure 3 shows contours of electrostatic potential in the plane $y = 0.0$ cm and particle orbits projected to the plane. The predicted transverse velocity at the exit is

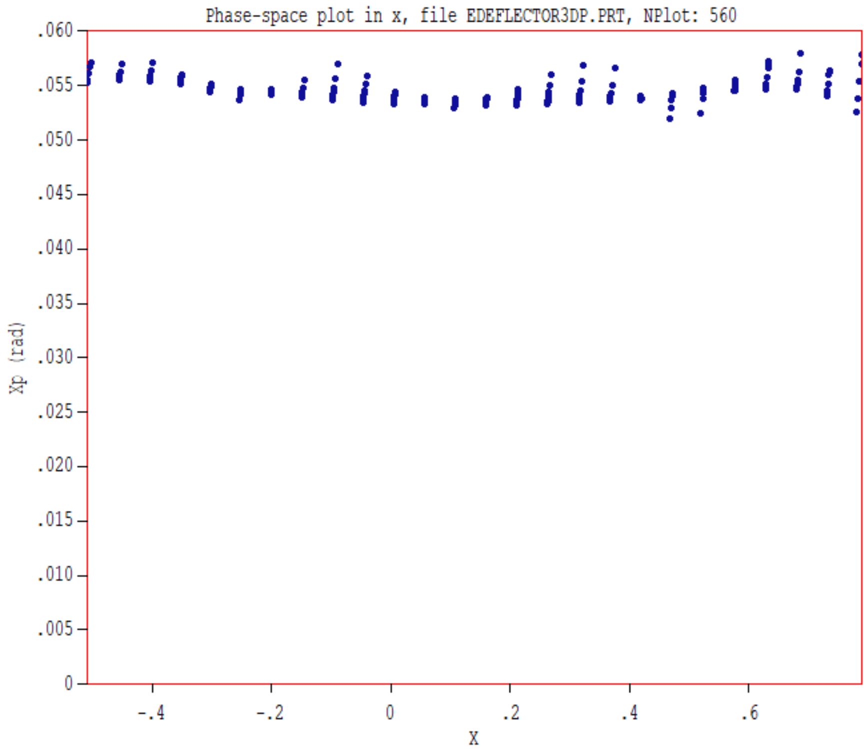

Figure 4: **GenDist** phase space plot of x-x' from the output file `EDeflector3DP.PRT`.

$$v_x \cong \frac{QE_x(L/V)}{M} = 1.78 \text{ m/s}. \tag{6}$$

The predicted exit angle is $\Delta\theta_x \cong v_x/V = 1.78/40.0 = 0.0446$ radian (2.56°). Figure 4 shows a phase space plot in x for the 560 test particles. Analysis of the angles gives $\Delta\theta_x = 0.05435 \pm 0.001$ radians. The value is somewhat higher than the simple prediction because of additional transverse fields in the entrance and exit regions. Finally, Fig. 5 shows the model particle distribution at the entrance and exit planes. The beam is uniformly displaced with little change in shape and size. Small deviations are discernable on the envelope.

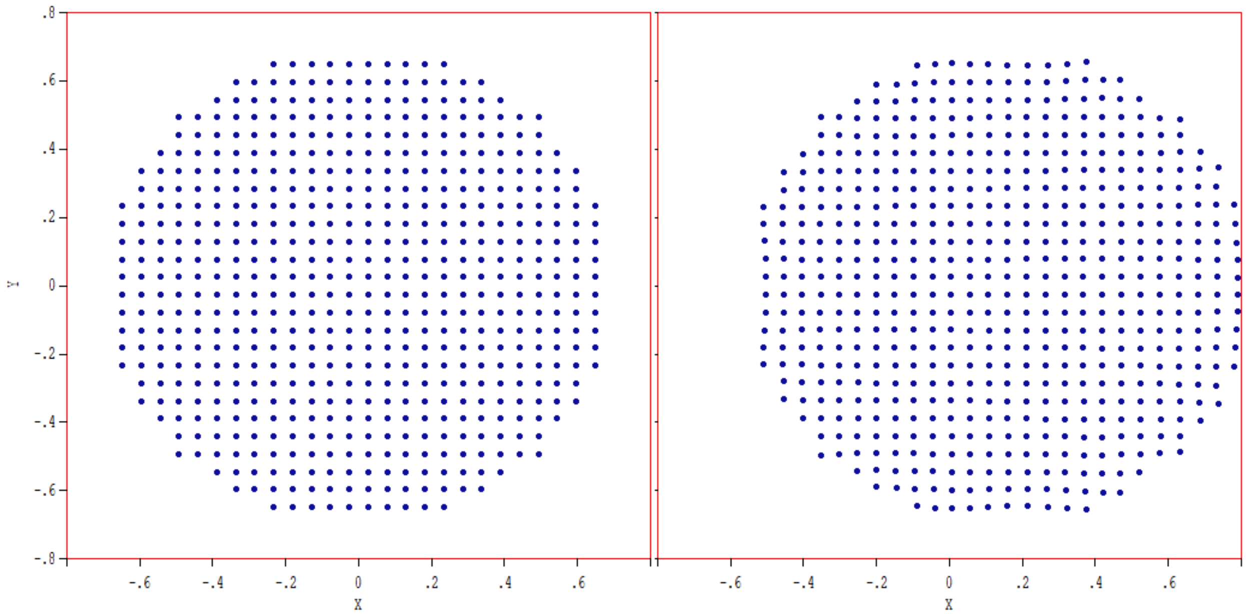

Figure 5: **GenDist** plots of particle distributions in the x-y plane at the entrance and exit planes ($z = \pm 2.5$ cm) from the output file `EDeflector3DP.PRT`.