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Two-dimensional space-charge-limited charged particle flow

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The maximum current density of non-relativistic charged particle flow across an infinite planar gap is described by the familiar Child law:

$$j_c = \left(\frac{4\epsilon_o}{9}\right) \sqrt{\frac{2q}{m}} \frac{V_o^{3/2}}{D^2}. \tag{1}$$

In the equation, V_0 is the applied voltage, m is the particle mass, q is the charge and D is the gap spacing (MKS units).

The 2D calculations of this report describe space-charge limited flow across a planar gap from finite emission areas (circles and slots). Under the assumption of uniform emission current density, the following analytic result was derived for a circular source by Lau[1]:

$$\frac{I}{I_c} = 1 + \frac{D}{4R}. \tag{2}$$

Here, I is the total emission current, R is the source radius and

$$I_c = j_c\ (\pi R^2). \tag{3}$$

The result was confirmed with particle-in-cell simulations by Luginsland, et.al.[2] under the same limiting assumptions in the range $D/R > 2$. The quantities in Eq. 2 represent an effective choice to display the results. Even though the current I may vary with R over many orders of magnitude, the quantity I/I_c remains close to unity. It represents the enhancement of flow in a two-dimensional geometry where the beam spreads over an area larger than the emission surface, reducing virtual cathode effects. Equation 2 also suggests the use of (R/D) as a scaling parameter in numerical calculations. If we perform calculations for a specific choice of Z, m, V_0 and D with variations of R, then the results can be applied to systems of any non-relativistic particles and any geometry by applying the scaling laws. The two-dimensional calculations have practical as well as academic significance. For example, we can predict the maximum possible extracted electron current from a laser spot on a planar photo-emitter.

This report describes numerically-exact results generated with the **Trak** beam program[3]. The code uses the ray-tracing technique[4] for high-accuracy simulations of steady-state, self-consistent charged particle flows. The new results extend the previous work in three areas:

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[1]Y. Y. Lau, Phys. Rev. Lett. **87**, 278301 (2001).

[2]J. W. Luginsland, Y. Y. Lau, R. J. Umstattd, and J. J. Watrous, Phys. Plasmas **9**, 2371 (2002).

[3]S. Humphries, J. Comp. Phys **125**, 488 (1996)

[4]W. B. Herrmannsfeldt, Stanford Linear Acc. Center, **SLAC-331**, 1988 and A. C. Paul, Lawrence Berkeley Lab, **LBL-13241**, 1982.

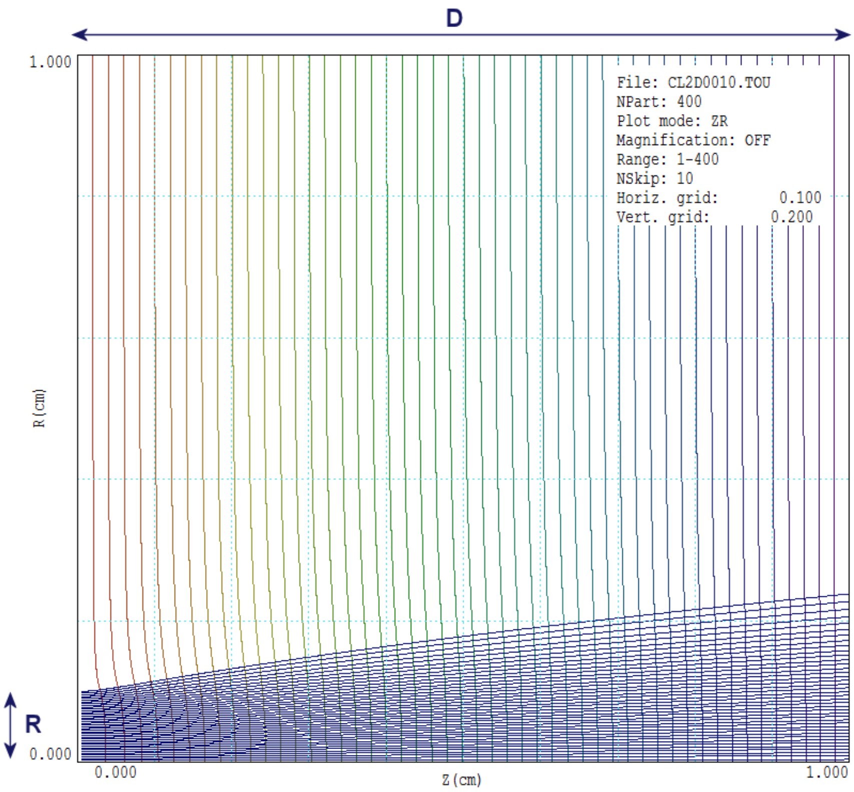

Figure 1: Calculated orbits and equipotential lines for $R = 0.1D$. One tenth of the orbits are plotted

- The condition of uniform emission is removed giving a self-consistent determination of current density variations over the source.
- The method applies in the range $D/R \gg 1$ (*i.e.*, small radius sources).
- The calculations give detailed information about the extracted beam quality.

The **Trak** calculations were performed for non-relativistic electrons with $D = 1.0$ cm and $V_0 = 5000$ V. I checked eight source radius values over the range $R = 5D$ to $R = 0.025D$. It was necessary to prepare optimized meshes for each value to maintain the number of ray traces at around 400 and to maintain an emission gap spacing small compared to R. Figure 1 shows the geometric parameters and calculated results for $R = 0.1D$. For clarity, only 10% of the traces are plotted. The curvature of the equipotential lines shows the effect of the beam space charge. Figure 2 shows the main result, the enhancement of extracted current over the simple application of the planar law of Eq. 3. The plot shows $(I/I_c - 1)$ determined by **Trak** along with the

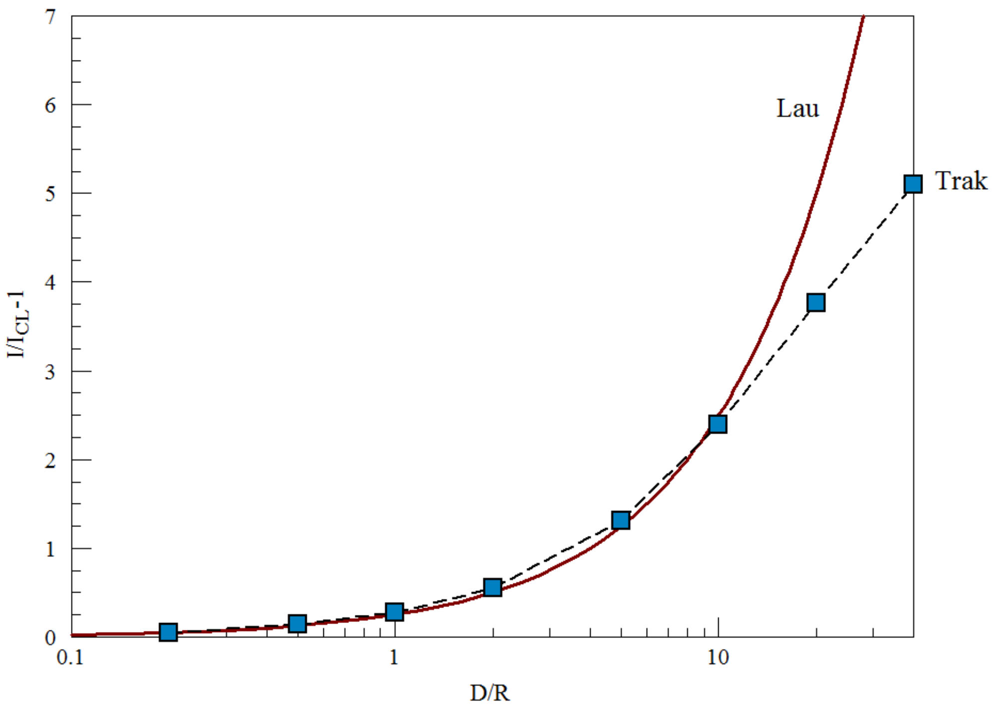

Figure 2: **Trak** calculation of $(I/I_c - 1)$ as a function of (D/R) for a source of radius R and an acceleration gap of width D. The solid line shows the prediction of Eq. 2.

Lau prediction, $(D/4R)$. Agreement is good for larger source radii, $R > 0.1D$ but deviates for small radius sources.

The difference probably results from the assumption of uniform emission current density in the analytic and particle-in-cell models. Figure 3 plots the current density at the emission surface determined by **Trak**. When the source radius is much larger than the gap width (Fig. 3a), the current density is close to the value 0.8238 A/cm^2 predicted by Eq. 1 over most of the source with enhancement at the edge. In contrast, at small radius (Fig. 3b), the current density exceeds the one-dimensional Child law prediction with considerable variation over the full source area.

The source radius also affects the quality of the beam. Figure 4 plots r' of trajectory as a function of r for large (Fig. 4a) and small (Fig. 4b) radius sources. Because of the non-linear transverse forces, the large radius beam has significant effective emittance, while the small source distribution follows almost a straight line. Hence, high perveance sources require focusing electrodes for a low-emittance beam. Although the small source gives better beam quality, keep in mind that the extracted current is much smaller (30.9 mA versus 3.30 A).

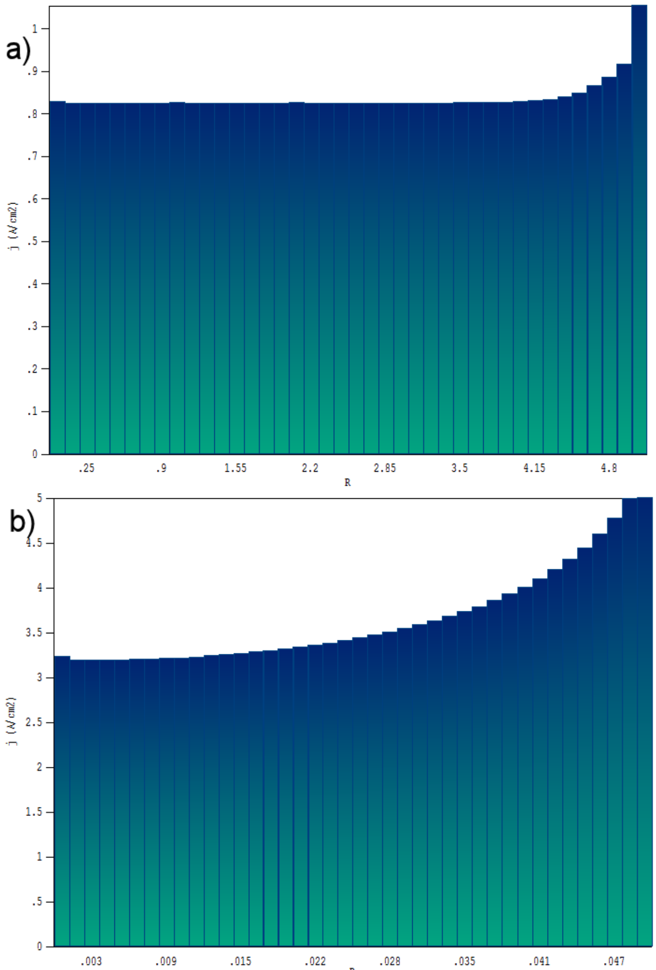

Figure 3: Emission current density as a function of radius for $a)$ $R/D = 5.0$ and $b)$ $R/D = 0.05$.

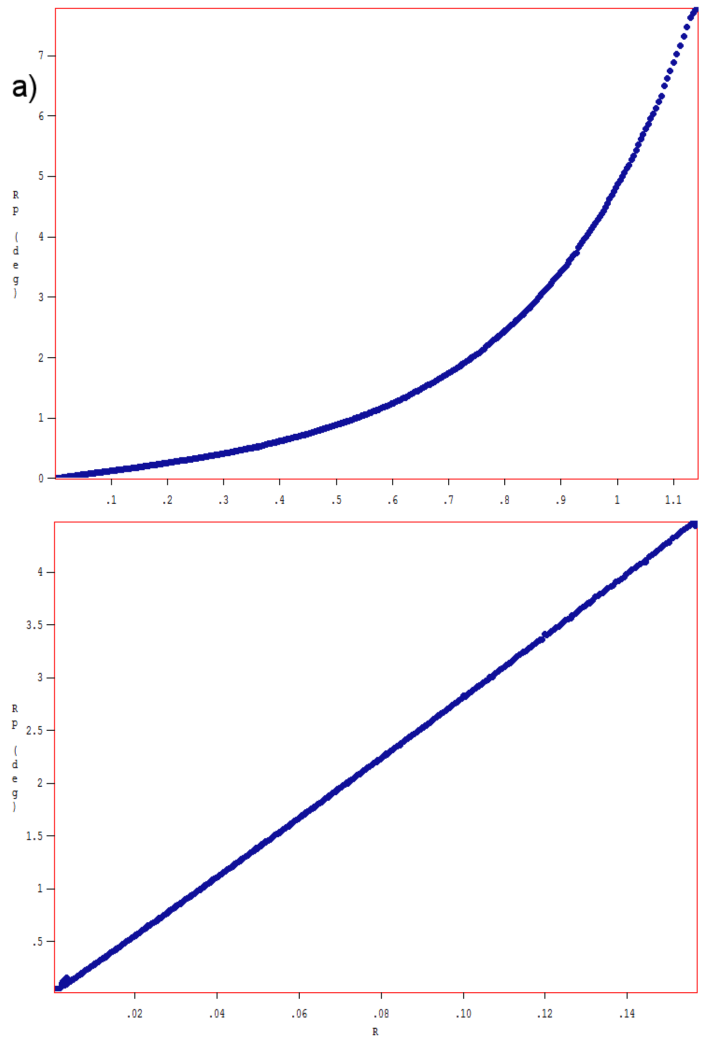

Figure 4: Radial phase space distribution (r' versus r) at the exit plane ($z = 1.0$ cm) for a) $R/D = 1.0$ and b) $R/D = 0.05$.

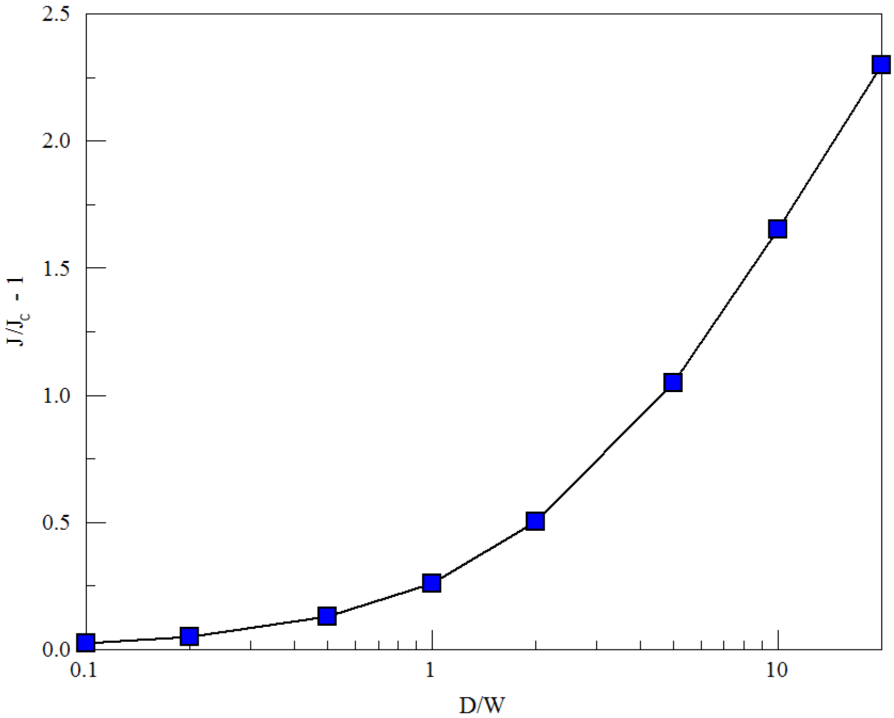

Figure 5: The quantity $(J/J_c - 1)$ as a function of (D/W) for a long slot of full-width W and an acceleration gap of width D. The quantity J is the linear current density current per length along the slot and $J_c = wj_c$.

Finally for completeness, Figure 5 shows the linear current enhancement calculated by **Trak** for a planar two-dimensional geometry. The quantity W is the full width of an emission slot of infinite length and again D is the width of the planar acceleration gap. Here, J is the linear current density (current per length along the slot) and $J_c = wj_c$.

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