Field Precision LLC

2D/3D simulations of microwave heating in resonant cavities

Stanley Humphries, Ph.D.

We recently added the capability to export three-dimensional power-density distributions calculated by **Aether** in the *RF* mode to **HeatWave** for thermal simulations. A user suggested an application – heating of dielectric objects in a microwave resonator. We carried out detailed calculations for this application to test the new code features and to confirm the existing capabilities of the two-dimensional codes **WaveSim** and **TDiff**. This tutorial illustrates how to set up coupled RF/thermal solutions and how to ensure the validity of results. The input files have been included in the **WaveSim/TDiff** and **Aether/HeatWave** example libraries.

Figure 1 shows the geometry for the benchmark solution. The system has cylindrical symmetry so we could make comparisons between the two- and three-dimensional results. A mode of type TM_{010} was generated in the cavity of radius $R_c = 15.0$ cm and length $D = 12.0$ cm. The theoretical frequency for the empty cavity is

$$ f = \frac{2.405c}{2\pi R_c} = 765.54 \quad \text{MHz.} \tag{1} $$

The dielectric object had radius $R_o = 5.0$ cm and length $D_o = 6.0$ cm. The electromagnetic mode was generated by a current density source in the shape of a thin plate that acted as a capacitive driver.

Before discussing the **WaveSim/TDiff** solutions, it is useful to review what we mean by heating in a resonant cavity. The assumption is that the power absorbed by the dielectric object does not significantly distort the resonant electric and magnetic fields. In other words, the total system has a high Q value. In this case, the field distribution is almost independent of the geometry of the driver; therefore, it is not necessary to make a detailed model of the power coupler (*e.g.*, coupling loop or waveguide), an impossible task with a two-dimensional code. The implication is that we must exercise care in choosing the electrical conductivity of the dielectric.

The element size in the **Mesh** input file (`HeatingDemo2D.MIN`) was relatively small for high resolution, 1.0 mm. A uniform element size was used throughout the solution volume to avoid the possibility of instabilities in the **TDiff** calculation. In a **WaveSim** *H*-type solution, the boundaries of the solution volume act as ideal conducting walls, so there is no need to define wall elements. Table 1 shows the **WaveSim** input file `HeatingDemo2D.WIN`. The source region has current density pointing in the z direction. In resonant solution, the amplitude of the source and resulting fields is arbitrary. We can normalize the fields to achieve desired power levels. There are three definitions for the dielectric controlled by comment-line symbols: 1) ideal vacuum, 2) ideal dielectric with relative dielectric constant $\epsilon'_r = 12.0$ and an absorbing dielectric with $\epsilon'_r = 12.0$ and $\epsilon_r" = 12.0 \times 10^{-6}$. With no dielectric, **WaveSim** gives a resonant frequency of 764.9 MHz. The frequency drops
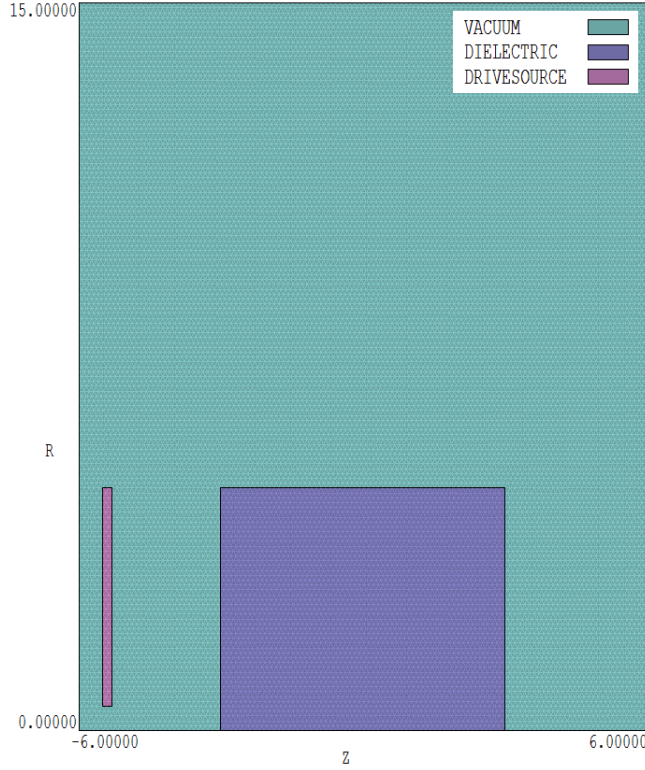
Figure 1: Geometry of the test calculation, dimensions in cm, cylindrical symmetry.

to 626.33 MHz with the addition of the dielectric. Figure 2 shows lines of applied electric field[1].

In **WaveSim**, imperfect dielectrics are defined by adding an imaginary part of the relative dielectic constant, ϵ_r". Power losses may result from conductivity or a phase lag in the dielectric response. In the case of resistivity losses, the conductivity and imaginary part of the dielectric constant are related by

$$\sigma = 2\pi f \epsilon_0 \epsilon_r ",\qquad(2)$$

where f is the mode frequency. The loss power density is related to the imaginary part of the dielectric constant and local electric field by

$$\overline{p} = 2\pi f \epsilon_0 \, \frac{\epsilon_r " |E^2|}{2}\qquad(3)$$

The time-averaged electric field energy density is

---

[1]Note that the total field amplitude (with contributions of the dielectric charge) inside the object is considerably reduced, as expected

Table 1: **WaveSim** input file `HeatingDemo2D.win`

```
Mesh Heating_Demo2D
Geometry = Cylin
DUnit = 100.0
Solution = H
Mode = Search
* Range = (700.0E6 800.0E6)
Range = (600.0E6 650.0E6)
FStep = 15
Tolerance = 1.0E-6
Probe = (0.0, 14.0)
* Vacuum
Mu(1) = 1.0
Epsi(1) = 1.0
* Dielectric
Mu(2) = 1.0
* Empty cavity
* Epsi(2) = 1.0
* Ideal dielectric
* Epsi(2) = 12.0
* Dielectric absorber
Epsi(2) = 12.0,1.2E-5
* Drive source
Mu(3) = 1.0
Epsi(3) = 1.0
Source(3) = (1.0, 0.0, 0.0)
ENDFILE
```
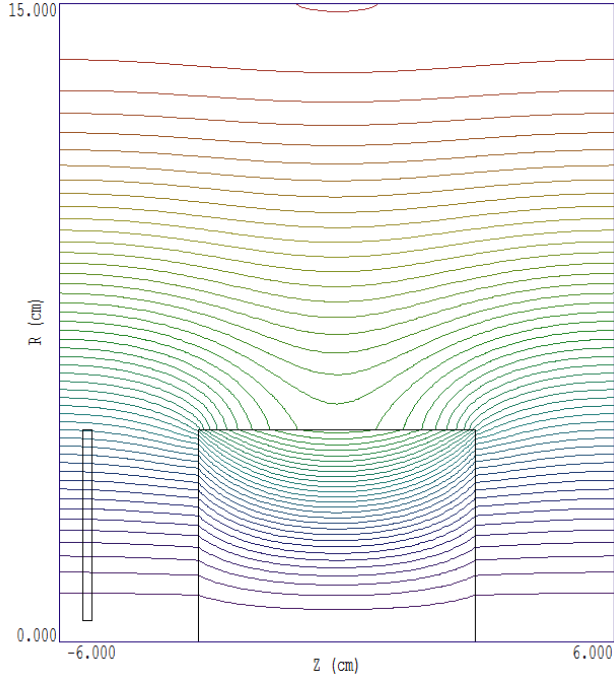
Figure 2: Solution with dielectric, lines of applied electric field.

$$\overline{u_e} = \epsilon_0 \ \frac{\epsilon_r' |E^2|}{4}. \tag{4}$$

Combining Eqs 3 and 4, the time-averaged power loss can be written in terms of the time-average electric field energy density:

$$\overline{p} = 4\pi f \ \left(\frac{\epsilon_r''}{\epsilon_r'} \right) \ \overline{u_e}. \tag{5}$$

The implication of Eq. 5 is that the system will have high Q if $\epsilon_r'' \ll \epsilon_r'$. Note in Table 1 that the ratio is 10^{-6}, so the condition of high-Q is well satisfied.

As expected, dielectric losses did not significantly affect the resonant frequency (f = 626.33 MHz). By chance, the field and power levels in the **WaveSim** solution were in a range of interest for a thermal calculation, so it was not necessary to re-normalize the field. The electric field magnitude on axis at $z = 0.0$ cm was 4.641 MV/m and the power density was 4.503 MW/m^3. The global electric field energy in the dielectric region was 0.1327 J and the absorbed power loss was 1.04452 kW, in agreement with the prediction of Eq. 5.

The next step was to use the **WaveSim** output as a source in a dynamic **TDiff** calculation. Table 2 shows the input file. The thermal calculation was
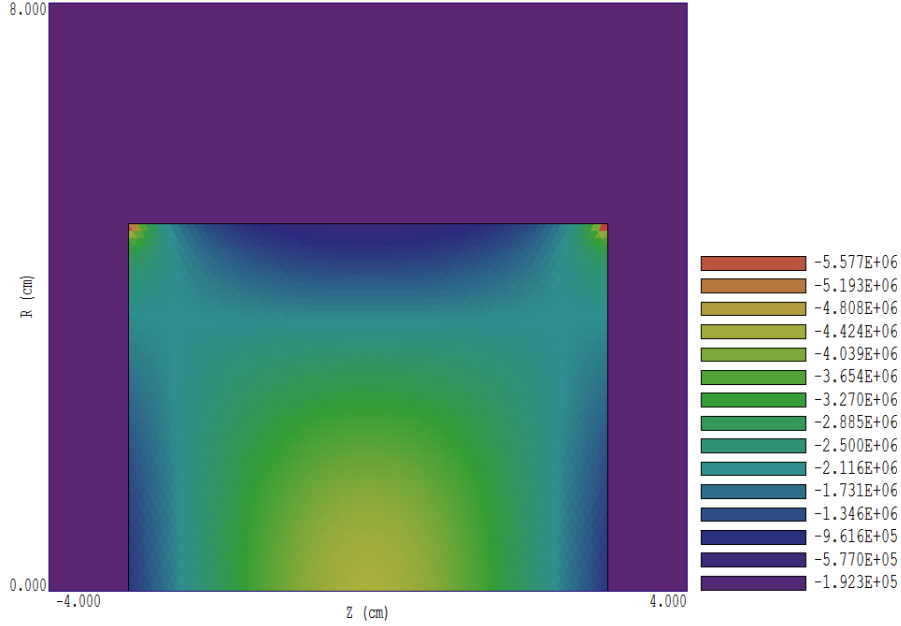
5

Figure 3: Absorbed power density inside the dielectric in W/m³.

performed on the same mesh as the **WaveSim** solution. The vacuum regions were assigned a low thermal conductivity, specific heat and mass so they had negligible effect on the thermal balance in the dielectric. Temperature distributions in vacuum were ignored. The line region on axis (Region 4) necessary for the **WaveSim** solution played no part in the **TDiff** calculation. It was assigned the material properties of vacuum to ensure that the nodes were treated as variable temperature.

The parameters for the dielectric were chosen arbitrarily to give a valid thermal solution: $C_p = 1200$ J/kg-°C and $\rho = 1300$ kg/m³. In the first run, the conductivity was set low ($k = 0.1$ W/m²-°C) to make a comparison with theory. For average power density $\bar{p}$, the predicted temperature rise with no thermal conductivity is

$$\Delta T = \frac{\bar{p}}{\rho C_p} \, \Delta t. \tag{6}$$

Using the **WaveSim** power density at the midpoint on axis,the predicted maximum temperature at the end of the 10 second run was 29.75°C.

The top of Fig. 4 shows the temperature distribution in the dielectric calculated by **TDiff** at 10.0 s. The temperature at the reference point ($z = 0.0$ cm, $r = 0.0$ cm) was 28.86 °C. In a second run, the conductivity was set to a high value, $k = 10.0$ W/m²-°C. In this case, the temperature profile was more diffuse and the value at the reference point was 25.60 °C.

6

Table 2: **TDiff** input file `HeatingDemo2D.tin`

```
* ---- CONTROL ----
Mesh = heating_demo2D
Mode = TVar
Geometry = Cylin
DUnit =   1.0000E+02
TMax =   1.1000E+01
DtMin =   1.0000E-03
DtMax =   2.0000E-02
Safety = 5.0
SourceFile heating_demo2D.wou
* ---- REGIONS ----
* Region 1: VACUUM
Cond(1) =   1.0000E-03
Cp(1) =   1.0000E+00
Dens(1) =   1.0000E+00
* Region 2: DIELECTRIC
Cond(2) =   1.0000E-01
* Cond(2) =   1.0000E+01
Cp(2) =   1.2000E+03
Dens(2) =   1.3000E+03
* Region 3: DRIVESOURCE
Cond(3) =   1.0000E-03
Cp(3) =   1.0000E+00
Dens(3) =   1.0000E+00
* Region 4: AXIS
Cond(4) =   1.0000E-03
Cp(4) =   1.0000E+00
Dens(4) =   1.0000E+00
* ---- DIAGNOSTICS ----
DTime =   1.0000E+01

EndFile
```
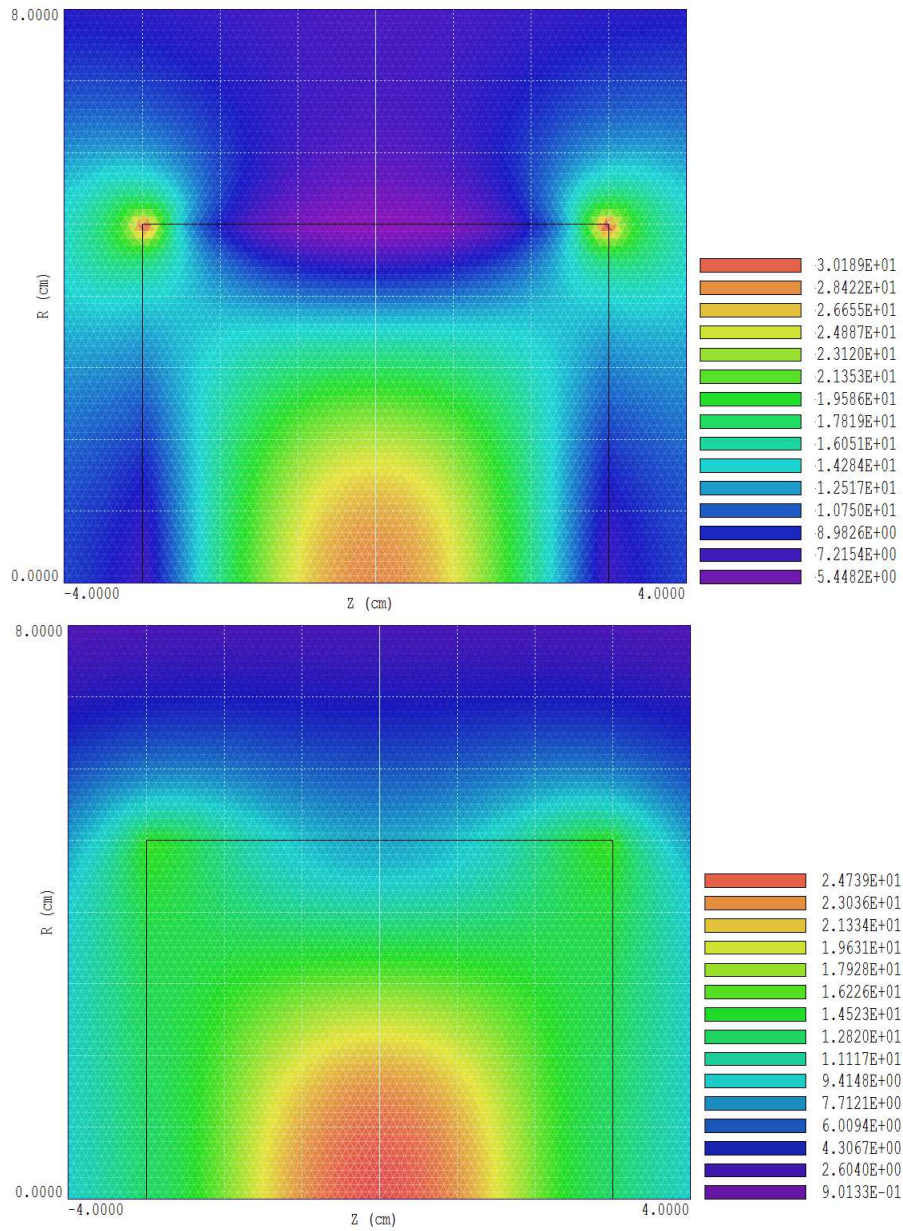
Figure 4: Temperature distributions calculated by **TDiff** with different dielectric thermal conductivities. Top: k = 0.1 W/m²-ºC. Bottom: Top: k = 10.0 W/m²-ºC.
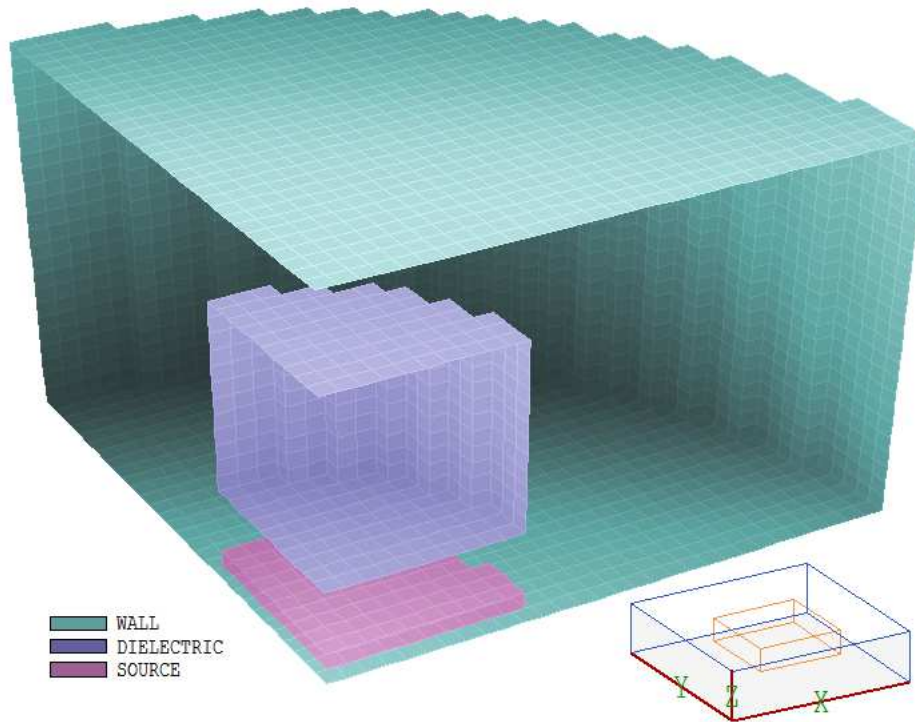
Figure 5: Mesh geometry for the **Aether** and **HeatWave** calculations showing one-quarter of the solution volume.

The same calculation was performed with **Aether** and **HeatWave** using the new capability to share power-density profiles. The solution used the same geometry and material properties as the two-dimensional calculation. In the **MetaMesh** input file `HeatingDemo3D.MIN`, an element width of 5.0 mm was used for a short run time. Therefore, we would not expect to resolve fine details like the field enhancement on the sharp edges of the dielectric (Fig. 2). Figure 5 shows a cutaway view of one quarter of the mesh. Note that a regular mesh is required for an **Aether** calculation. Despite the box elements and the relatively coarse zoning, both the microwave and thermal solutions were close to the two-dimensional results. In contrast to **WaveSim**, in **Aether** an outer region of elements with the properties of a perfect conductor is necessary to define the walls of the cylindrical cavity. The inner surface of this region is shown in blue in Fig. 5. The dielectric (violet) and drive region (red) had the same shape as those in the two-dimensional calculation.

Another difference in **Aether** is that the solution must be performed in two parts: 1) identification of the resonant frequency and 2) calculation of the field distribution at that frequency. Table 3 list the **Aether** input file for the resonance solution. The calculated frequencies were 766.7 MHz with no

Table 3: **Aether** input file `HeatingDemo3DRes.win`

```
* ---- CONTROL ----
Mode = RES
Mesh = Heating_Demo3D
DUnit =  100.0
Freq  0.60E9 0.30E9
Source 3 0.0 0.0 1.0
Parallel 6
* ---- REGION PROPERTIES ----
Metal(1)
Vacuum(2)
Epsi(3) 12
Sigma(3) 4.178.8E-7
Vacuum(4)
* ---- DIAGNOSTICS ----
Probe = 14.500  0.000  0.000 Hy

EndFile
```

dielectric and 626.12 MHz with the dielectric (both ideal and conducting). The agreement with **WaveSim** was better than 0.25%. Another difference is that **Aether** requires specification of the conductivity rather than imaginary part of the dielectric constant. The value $\sigma = 4.178.8 \times 10^{-7}$ S/m was determined from Eq. 2 with $f = 626.12$ MHz.

The **Aether** input script for the second solution had the control entries

```
Mode = RF
Freq   6.261167E+08
NPeriod  15   3   10
```

An *RF* mode calculation differs from a *RES* mode calculation in two ways: 1) it is performed at specific drive frequency and 2) Aether creates a data file of the field and power-deposition distributions. This data may be transferred to **HeatWave**. The parameters in the `NPeriod` command have the following meaning. Run the time-domain solution for 15 RF periods. The current in the drive region should start smoothly over 3 periods and stop after 10 periods to let the fields come to equilibrium. At the end of the run, **Aether** determines the amplitude and phase of field quantities to find complex number values for the output file.

The amplitude of the drive current was set to the arbitrary value $J_z = 1.0$ A/m^2. Therefore, it was necessary to normalize the field for a comparison
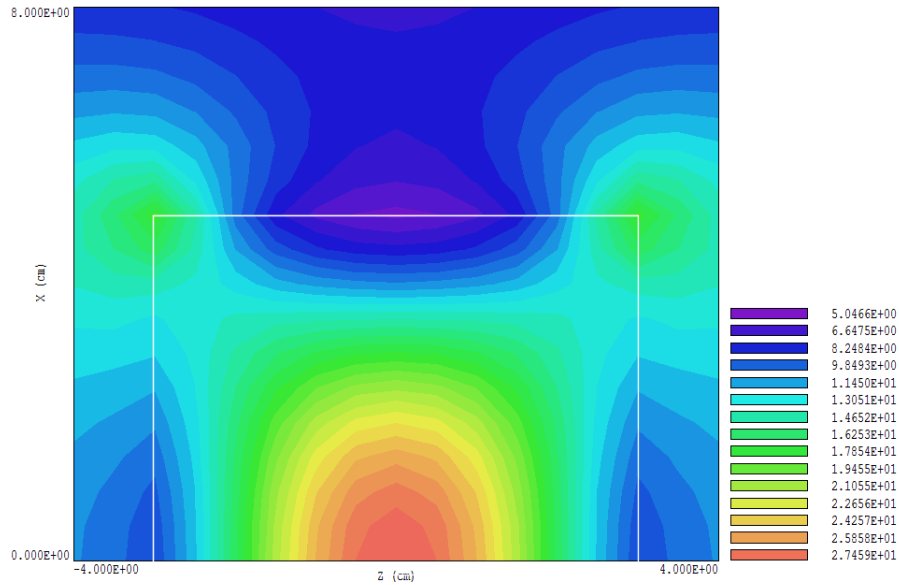
Figure 6: Temperature distribution calculated by **HeatWave** for a thermal conductivity k = 0.1 W/m²-ºC.

with the **WaveSim** results. The **Aerial** postprocessor showed that the electric field at the reference point was $E_z = 4.2384$ V/m, compared to the WaveSim result of 4.641×10^6 V/m. A factor of 1.0844×10^6 was entered in the *Analysis/Normalize* command of **Aerial** and the result was saved as the file `HeatingDemo3DNorm.aou`.

Table 4 shows the **HeatWave** input file. The new capability to import **Aether** power profiles appears in the command

```
SourceFile HeatingDemo3DNorm.aou.
```

Figure 6 shows the resulting temperature distribution in the dielectric at 10 seconds for thermal conductivity k = 0.1 W/m²-ºC. The temperature at the reference point was 28.33 ºC, within 2% of the **TDiff** prediction. At high conductivity (k = 10.0 W/m²-ºC) the value was 25.17 ºC, comparable to the **TDiff** result of 25.60 ºC.

Table 4: **HeatWave** input file `HeatingDemo3D.win`

```
* ---- CONTROL ----
Mesh = heating_demo3D
Mode = Dynamic
DUnit =    1.000000E+02
TMax =    1.100000E+01
DtMin =    1.000000E-03
DtMax =    5.000000E-03
SourceFile HeatingDemo3DNorm.aou
* ---- MATERIAL PROPERTIES ----
* Material 1
Cond(1) =    1.000000E-03
Cp(1) =    1.000000E+00
Dens(1) =    1.000000E+00
* Material 2
Cond(2) =    1.000000E-03
Cp(2) =    1.000000E+00
Dens(2) =    1.000000E+00
* Material 3
* Cond(3) =    1.000000E+01
Cond(3) =    1.000000E-01
Cp(3) =    1.200000E+03
Dens(3) =    1.300000E+03
* Material 4
Cond(4) =    1.000000E-03
Cp(4) =    1.000000E+00
Dens(4) =    1.000000E+00
* ---- REGION ASSIGNMENTS ----
* Region 1: WALL
Region(1) =    1
* Region 2: VACUUM
Region(2) =    2
* Region 3: DIELECTRIC
Region(3) =    3
* Region 4: SOURCE
Region(4) =    4
* ---- DIAGNOSTICS ----
DTime =    1.000000E+01
EndFile
```