

Calculating klystron output cavity loading

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Figure 1: Cut-away view of a klystron output cavity coupled to a 75 Ω coaxial transmission line.

The full three-dimensional resources of **Aether** are applied to calculate loading of the output cavity of a hollow-beam klystron. The goal is to optimize the loop coupled to an output transmission line to achieve a target value of loaded Q. Figure 1 shows the simulation geometry, a reentrant output cavity for a high-power klystron. The cavity has an outer radius of 6.0 cm. The RF power is extracted through a 75 Ω vacuum coaxial transmission line with outer radius $R_o = 2.0$ cm and inner radius $R_i = 0.6$ cm. For the study it is assumed that there is an ideal vacuum window at a downstream point in the line so that there is no reflected power. The calculation represents a 4.0 cm length of the line with an absorbing boundary layer at the end. There are three primary quantities to find: 1) the resonant frequency of the structure, 2) the peak electric field levels and 3) the Q factor determined by output loading.

1 Resonant frequency calculation

The first task was to find the resonant frequency of the loaded cavity. To check the 3D model, a 2D calculation using our **WaveSim** code was performed for a cylindrically-symmetric cavity without the transmission line. The test was definitive because **WaveSim** used completely different numerical methods: triangular elements and a solution of the Helmholtz equation



Figure 2: Cavity geometry and electric field lines. Two-dimensional **WaveSim** calculation.

by a direct matrix inversion. Figure 2 shows a z-r plot of the cavity geometry and electric field lines of the fundamental mode. The beam transport line was below cutoff; therefore, the boundaries at $z = \pm 3.0$ cm did not affect the calculation. With an element size of 0.05 cm, the calculated TM₀₁₀ frequency was 1.409 GHz.

The vectors comprising the outline of the cavity wall were transferred to **MetaMesh** to generate a three-dimensional representation with an element size of 0.1 cm. The mesh had 627,669 elements. The strategy in **MetaMesh** was to fill the solution volume with metal elements and then to carve out the cavity using the outline created in the **Mesh** drawing editor. The resonator simulations involved a large number of metal elements, but there was no time penalty because **Aether** does not update field values inside metal regions. A small region with a drive current density j_z was located on axis (Fig. 1) to limit the response to modes of type TM_{010} . With excitation in the frequency band 0.0 to 3.0 GHz, **Aether** detected a single resonance at 1.402 GHz, within 0.5% of the **WaveSim** result.

The next step was to find the resonant frequency of the full system with the terminated transmission line. Table 1 shows the contents of the **Aether** script. The *Source* command defined a uniform current in the z direction in the drive region (Region 4). Region 1 comprised the cavity wall and the inner and outer conductors of the transmission line. Region 2 was the Table 1: Aether script to find the resonant frequency of the full system.

```
* ---- CONTROL ----
Mode = RES
Mesh = KLYS_FREQ3D
DUnit = 100.0
Freq 1.5E9 3.0E9
Source 4 0.0 0.0 1.0
* ---- REGION PROPERTIES ----
Metal(1)
Vacuum(2)
AbsLayer(3) 0.10
Vacuum(4)
* ---- DIAGNOSTICS ----
Probe = -5.50 0.00 0.00 Hy
EndFile
```

cavity volume and Region 3 was the line terminator (with thickness 0.1 cm). A probe was located at radius 5.50 cm across from the coupler. Figure 3 shows the Fourier transform of the probe signal near the resonance. The inclusion of the transmission line coupler reduced the cavity volume and consequently shifted the frequency up about 0.9% to 1.4209 GHz. The width of the response gave a rough estimate of the quality factor, $Q \sim 24.5$.

2 Mode field calculation

The resonant frequency $f_0 = 1.4209$ GHz was used in the final *RF*-mode calculation to find the field distribution and an accurate value of Q for the driven cavity. The **Aether** script contained the following lines:

Freq = 1.4209E9 NPeriod = 40 2

The relatively large number of RF periods was necessary to ensure that the fields reached equilibrium before conversion to phasor form. Some care must be taken to represent drive currents in a loaded RF solution. In this case, the drive was an annular beam with outer radius 0.5 cm and inner radius 0.3 that extended along the length of the transport tube. Figure 4 shows the mesh. The harmonic component of current at frequency f_0 in the bunched beam had amplitude $I_0 = 100$ A. The discrete representation of the beam cross-section included 48 elements, each with area 10^{-6} m². The current density to generate 100 A was $j_z = 2.083 \times 10^6$ A/m².



Figure 3: Frequency response of the loaded klystron cavity.



Figure 4: Annular beam drive current (orange) for the RF field solution.



Figure 5: Plot of $H_y(t)$, monitor at a radius r = 5.5

Figure 5 shows a plot of $H_y(t)$ at the probe position, confirming that the solution had reached a steady state. The run time was 1 hour and 20 minutes. One method to calculate cavity loading is to use the energy and power integrals recorded in the listing file. The quality factor is given by

$$Q = \frac{2\pi f_0 U}{P},\tag{1}$$

where U and P are time-averaged values of the electromagnetic energy in the cavity and power dissipated in the absorbing layer. The values determined in the **Aether** solution were U = 0.0485 J and P = 14.53 MW. Substitution in Eq. 1 yields Q = 29.8. The Q value can also be determined from the signal envelope in Fig. 5. The theoretical variation is

$$H_y(t) = H_y(\infty) \left[1 - \exp\left(-\frac{\pi f_0 t}{Q}\right) \right].$$
(2)

Measurements of the signal using the **Probe** utility imply that $Q \cong 31.5$.

The final activity is to inspect the mode fields. Figure 6 shows the variation of $|\mathbf{E}|$ over the x-y plane at z = 0.0. Here, the amplitude symbol refers to the peak value in time of the sum of the spatial components of the electric field. Note that the uniform value in the transmission line indicates a wave traveling to the right in the transmission line with no reflection at the absorbing layer (VSWR = 0). Using the *Line integral* command in **Aerial**, the voltage in the transmission line was determined to be $V_L = 45.2$ kV at a phase of 240°. The corresponding power flux is



Figure 6: Variation of $|\mathbf{E}|$ in the plane z = 0.0 cm.

$$P = \frac{V_L^2}{2Z_0} = 13.62 \text{ MW.}$$
(3)

The peak electric field value in the solution of 2.26 MV/m occurred on the tip of the smaller nose. A line integral $\int \mathbf{E} \cdot \mathbf{dl}$ across the axis of the cavity gave a cavity voltage $V_c = 295.4$ kV at 180°. The predicted beam power is $P = I_0 V_c/2 = 14.8$ MW. Within the accuracy of the line integrals, the three methods for estimating the RF power are consistent.