Field Precision

Numerical modeling of energy extraction from resonant cavities

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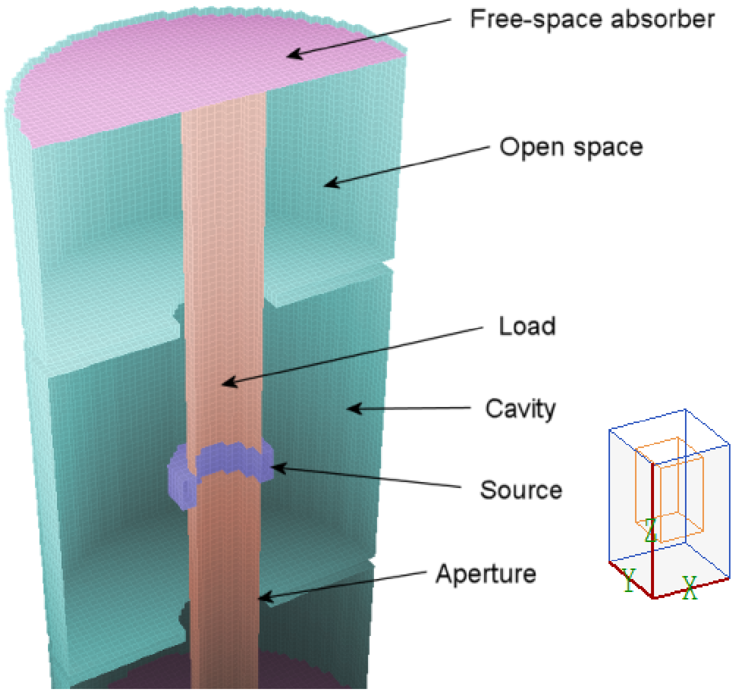

Figure 1: Open cylindrical resonant cavity.

Resonant cavities can act as voltage transformers, transferring power at high impedance to a transmission line or waveguide at low impedance. This tutorial describes how to model the process with **Aether**.

To start, we will find the properties of the example resonator. Figure 1 shows the geometry. We will work with the $\mathtt{TM}_{010}$ mode which produces a maximum electric field E_z along the axis. The vacuum cavity section has radius $r = 50.0$ mm and height $h = 100.0$ mm with openings of radius 15.0 mm at each end. The apertures allow access for a power source such as an electron beam. The opening is small enough to prevent radiative loss. Voids of height 58.0 mm with free-space absorbing boundaries of thickness 2.0 mm above and below the cavity accomodate the fringe fields. A cylindrical region of radius $r_l = 10.0$ mm that extends the length of the calculation can be assigned a non-zero conductivity. Finally, there is a small annular source carrying axial current to excite the resonance. The full height of the assembly is included to include asymmetric drives in following calculations. The theoretical frequency for a closed cavity is $f = 2.405c/2\pi R = 2.297$ GHz.

I performed initial calculations to find the resonance frequency of the open cavity. Figure 2 shows the **Aether** control script displayed in **ConText** with syntax highlighting. The `Res` mode search was centered near the expected frequency. The pulsed source current was parallel to the cav-

```
* ---- CONTROL ----
Mode = Res
Mesh = CavityProperties
DUnit =    1.00000E+03
Freq =    2.50000E+09  2.00000E+09
Source(3) =    0.00000E+00  0.00000E+00  1.00000E+00
Parallel 4
* ---- REGION PROPERTIES ----
Metal(1)
Epsi(2) =    1.00000E+00
Mu(2) =    1.00000E+00
Epsi(3) =    1.00000E+00
Mu(3) =    1.00000E+00
AbsLayer(4) = 2.0
Epsi(5) =    1.00000E+00
Mu(5) =    1.00000E+00
Sigma(5) =    0.00000E+00
* Sigma(5) =    5.00000E-02
* ---- DIAGNOSTICS ----
History =    0.00000E+00  0.00000E+00  0.00000E+00

EndFile
```

Figure 2: **Aether** script for the resonance search. Region associations: 1 (cavity wall), 2 (vacuum), 3) (drive source), 4 (absorber) and 5 (load region).

ity axis (z direction). I placed a history probe at the center of the cavity. The load region was assigned zero conductivity or a value of $\sigma = 0.05$ S/m. This value corresponded to a resistance across the cavity of approximately $R_c \cong h/\sigma\pi r_l^2 = 6.37$ kΩ.

Figure 3 shows the result, a Fourier transform of the residual fields after a pulse excitation. The resonance frequency of $f_0 = 2.305$ GHz was somewhat higher than the closed-cavity value because of the reduced capacitance from the presence of the apertures. The resonance for $\sigma = 0.0$ S/m (Fig. 3a) had a narrow width caused by discretization effects. As expected, the resonance width with loading (Fig. 3b) was broader – the frequency width at the 0.707 points was $\Delta f \cong 0.12$ GHz. The implied quality factor is $Q = f_0/\Delta f \cong 19.2$. We can get a more accurate estimate of the cavity Q from an RF mode calculation. The control parameters were changed to

```
Mode = RF
Mesh = CavityProperties
DUnit =   1.00000E+03
Freq =   2.305E9
NPeriod =  40 3 30
Parallel 4
```

The `NPeriod` command specifies that 1) the simulation extends over 40 RF cycles, 2) the drive current starts smoothly over 3 cycles to minimize high frequency noise and 3) the drive current is turned off after 30 cycles.

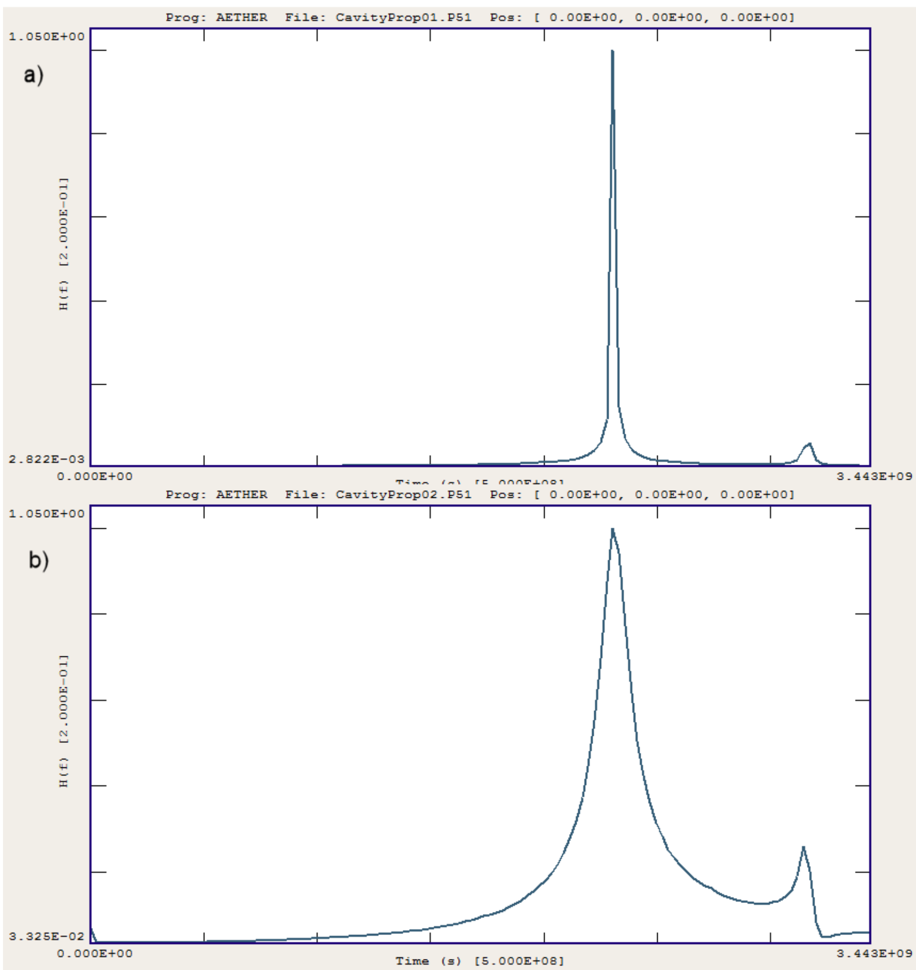

Figure 3: Resonance response of the cavity: $a)$ $\sigma = 0.00$ S/m, $b)$ $\sigma = 0.05$ S/m.

Figure 4 shows measurements of E_z at the cavity center. The top trace shows the signal with no loading. The signal increases without limit during the drive period and settles into an equilibrium when the drives stops at 30 cycles. The signal confirms that energy loss through the apertures is small. This run demonstrates the standard **Aether** procedure to find the field pattern of resonant structures. Figure 5 shows spatial variations of the amplitudes of the electric and magnetic fields at 40 cycles, close to those of the ideal $\mathtt{TM}_{010}$ mode.

With loading, the cavity electric field approaches a limiting value during the drive and decays after 30 cycles (Fig. 4*b*). The quality factor is also defined as

$$Q = 2\pi \, \frac{\text{Stored energy}}{\text{Energy lost per cycle}}. \tag{1}$$

Figure 6 shows Q values derived from the peak amplitudes of the signal in Fig. 4*b*. After initial transients, the signal decay implies a Q value close to 20. The final (and most accurate method) to find the quality factor is to use energy quantities in the **Aether** listing file (`ALS`) calculated at the run ending:

```
Energy and power volume integrals
Global quantities
   Electric field energy:   7.51293E-11 (J)
   Magnetic field energy:   6.69963E-11 (J)
   Total field energy:   1.42126E-10 (J)
   Resistive power dissipation:   1.06452E-01 (W)
```

Inserting values into Eq. 1 gives $Q = 19.35$.

With a thorough understanding of the cavity properties, we can address the topic of driven assemblies. I will start with the coaxial transmission line shown in Figure 7. The vacuum line has inner radius $r_i = 4.0$ mm and outer radius $r_o = 10.0$ mm. The impedance is

$$Z = \sqrt{\mu_0/\epsilon_0} \, \ln(r_o/r_i)/2\pi = 55.0 \ \Omega. \tag{2}$$

Following the discussion in Chapter 2 of the **Aether Tutorial Manual**, the line ends in a matched termination, an absorbing layer 2.0 mm thick. The line is connected to the cavity by a loop of radius 10.0 mm orientated to intercept H_θ of the $\mathtt{TM}_{010}$ mode. I recalculated the resonant frequency to include the effect of the transmission line using the same approach as above. The $\mathtt{TM}_{010}$ mode frequency was 2.325 GHz, somewhat higher because of the volume occupied by the loop. A narrow width implied that the assembly with transmission line loading had a relatively high Q value.

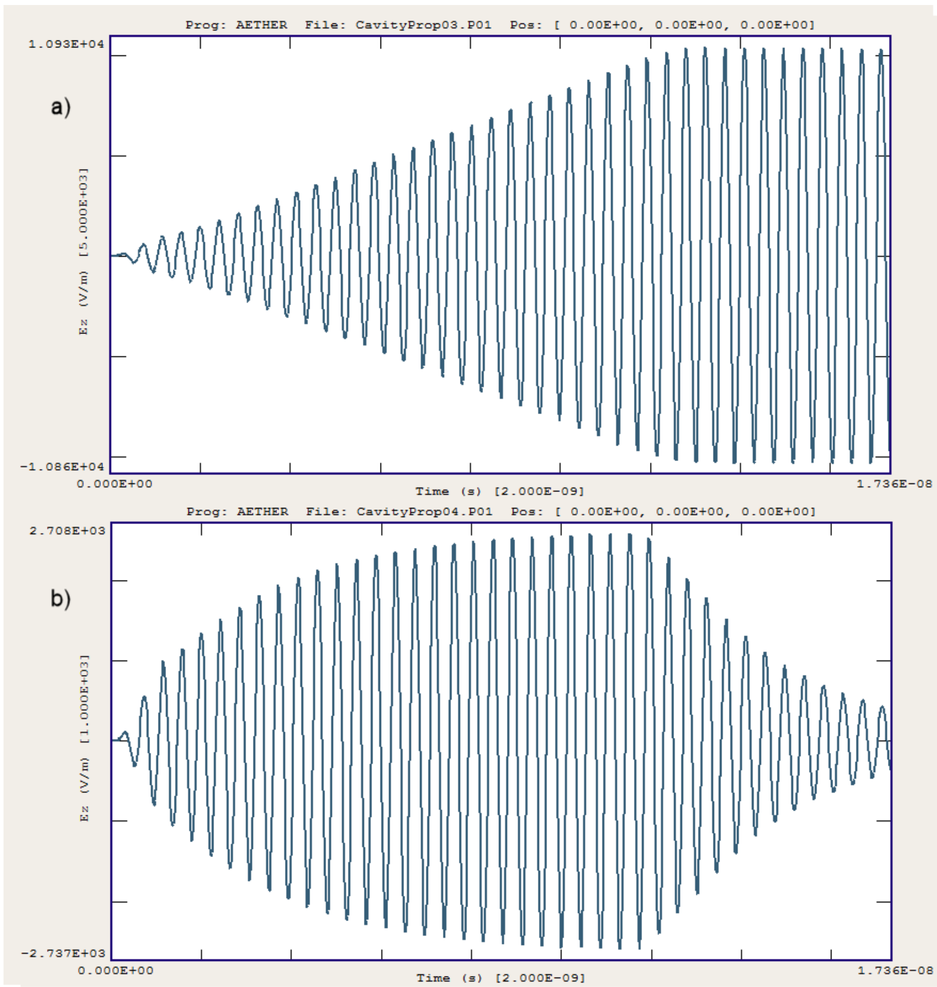

Figure 4: Measurements of E_z at the cavity center: *a*) $\sigma = 0.00$ S/m, *b*) $\sigma = 0.05$ S/m.

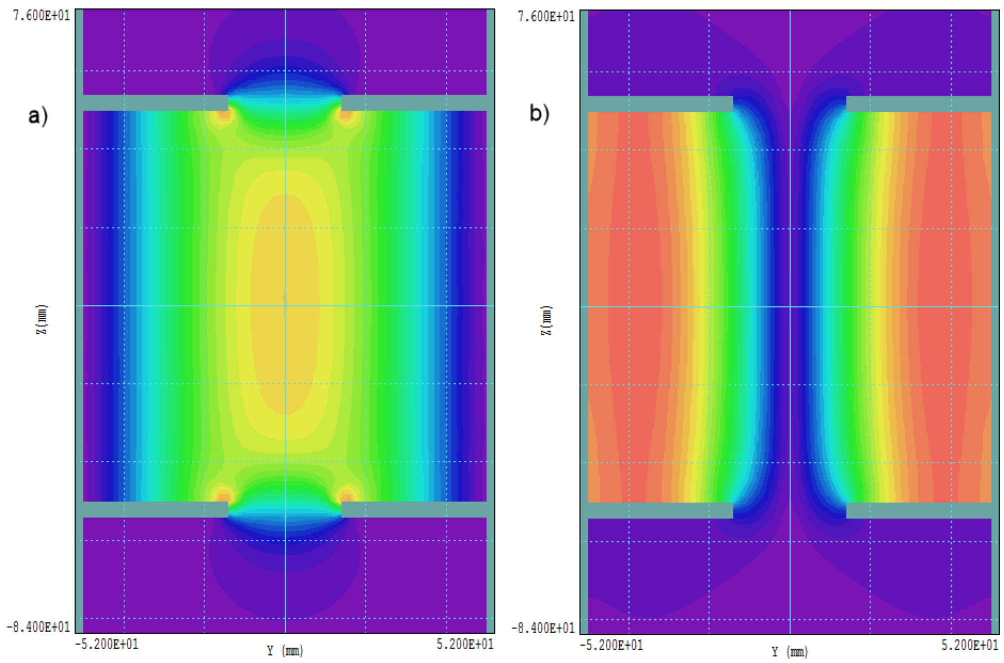

Figure 5: Field variations in the plane $x = 0.0$ mm with no cavity loading: a) $|E|$ V/m, b) $|H|$ A/m.

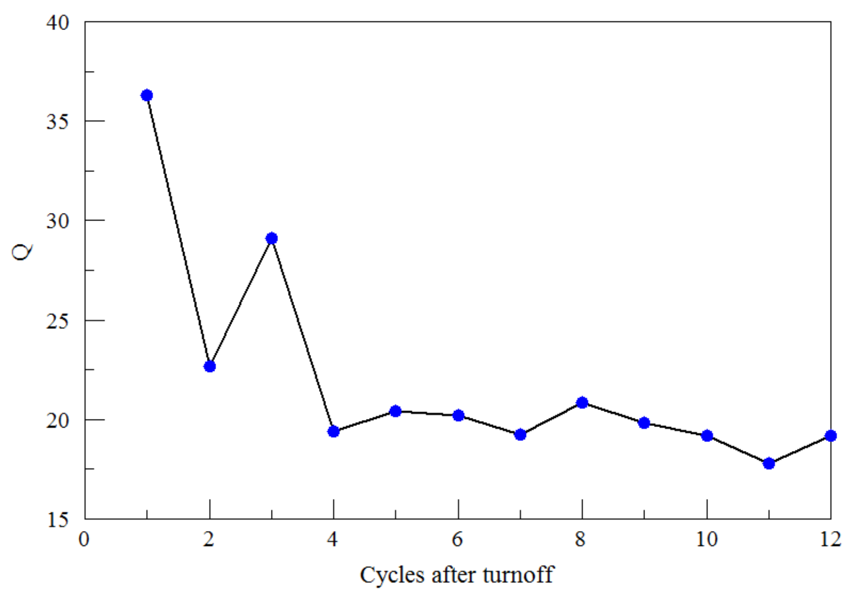

Figure 6: Plot of $2\pi E_z^2/\Delta(E_z^2)$ for the probe signal of Fig. 4b.

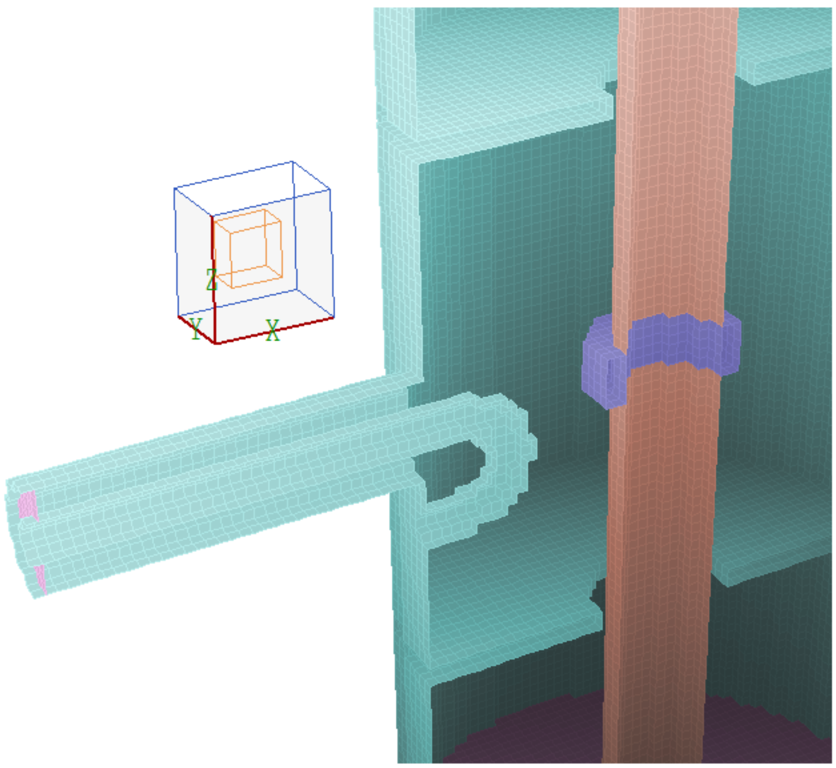

Figure 7: Resonant cavity with a vacuum transmission line connected by a coupling loop.

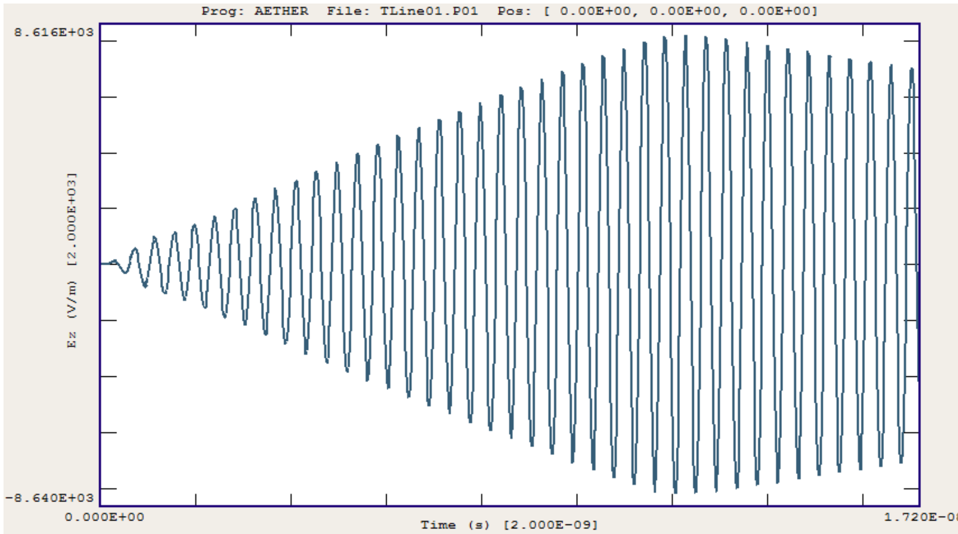

Figure 8: RF mode calculation with the attached transmission line, record of E_z at the center of the cavity.

I then used the resonant frequency in an RF mode calculation. Again, the in-cavity annular current source generated the fields. The current was active only over the first 30 cycles. Figure 8 shows a measurement of E_z at the cavity center. The field increased approximately linearly followed by a slow decay, confirming a high value of Q for the structure. An inspection of energy values recorded the run listing file gave a power dissipation in the line termination of 3.379 W and a cavity energy of 4.885×10^{-8} J, implying a quality factor of 211.2. The RF field variations were calculated at the end of the run. Figure 9 shows the distribution of $|\mathbf{E}|$ in the symmetry plane $y = 0.0$ mm. The $\texttt{TM}_{010}$ mode in the cavity was slightly distorted with a pure traveling `TEM` wave in the transmission line.

The other important quantity to characterize the coupler is the voltage stepdown ratio. The transmission line voltage is given by

$$V_l = \sqrt{2PZ_o} = 19.8\text{V}. \tag{3}$$

I define the cavity voltage V_c as the integral of E_z on the axis over the cavity length. The **Aerial** scan of $|\mathbf{E}|$ along the cavity axis (Fig. 10) gives

$$Vc = 7078.0 \times 0.101 = 714.2 \text{ V}. \tag{4}$$

The voltage stepdown ratio is 1.0/36.1.

I will conclude with an example of waveguide coupling. Figure 11 shows the geometry. A rectangular waveguide connects to the cavity through a rectangular slot. The waveguide carries the $\texttt{TE}_{10}$ mode with E_z parallel to the

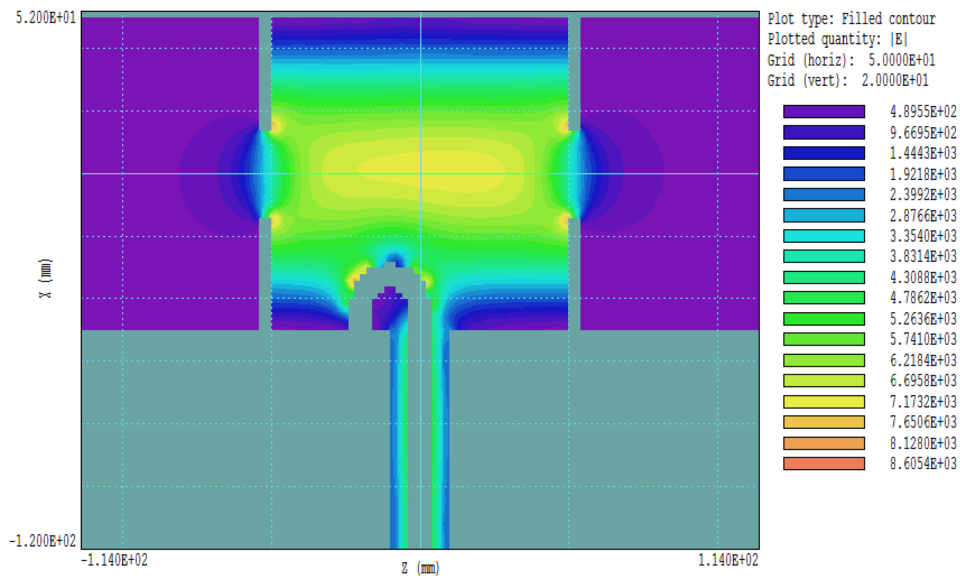

Figure 9: Plot of $|\mathbf{E}|$ in the plane $y = 0.0$ mm for the resonant cavity connected to a coaxial transmission line.

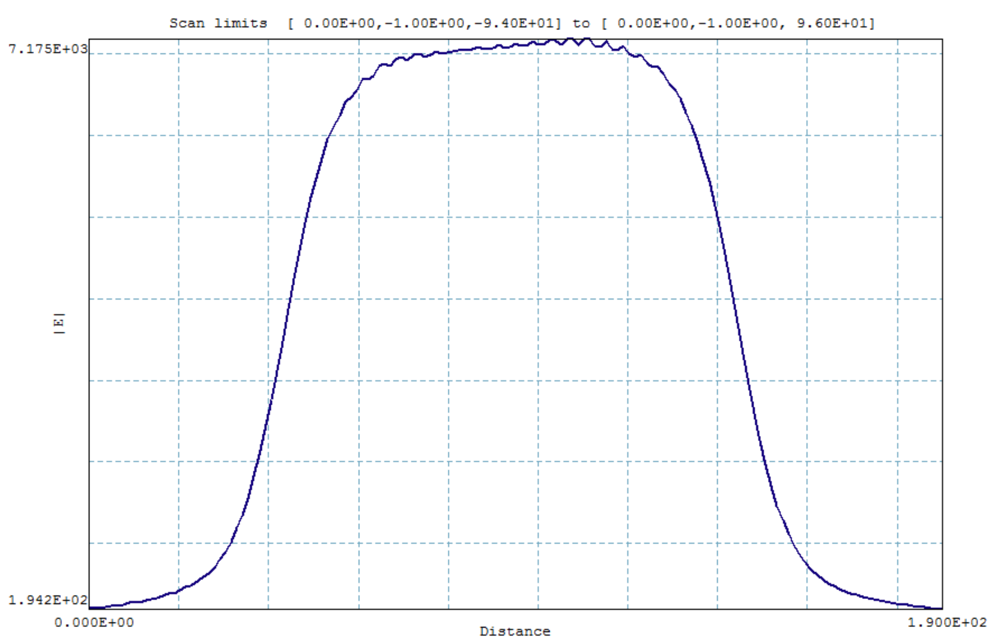

Figure 10: Scan of $|\mathbf{E}|$ along the cavity axis.

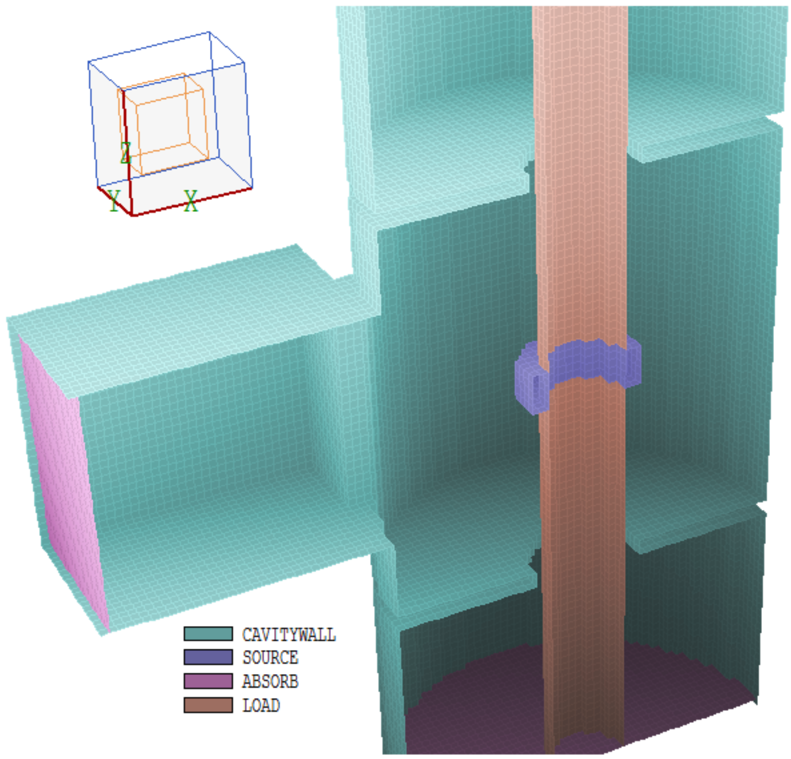

Figure 11: Resonant cavity driving a waveguide.

cavity axis. Coupling is primarily through shared magnetic fields. Chapter 9 of the **Aether Tutorial Manual** discusses techniques to model this mode. The waveguide has dimensions $a = 80.0$ mm and $b = 60.0$ mm. The cutoff frequency,

$$f_c = \frac{c}{2a} = 1.875 \times 10^9 \text{ Hz}. \tag{5}$$

is below the expected cavity resonance. The line terminates with a 2.0 mm thick absorbing layer. As discussed in the tutorial manual, the conductivity of the absorbing layer should be reduced by a factor of v_g/c, where v_g is the group velocity. At frequency 2.3 GHz, the ratio is 0.6. The slot with dimensions $a = 40.0$ mm and $b = 60.0$ mm gives strong coupling relative to the previous example. In this case, the voltage stepdown ratio is smaller but the cavity can drive a line with lower impedance.

An initial `Res` mode run gives a $\mathtt{TM}_{010}$ mode frequency 2.272 GHz. As before, the `RF` mode calculates extends over 40 cycles, with the cavity source current active for the first 30 cycles. A plot of $E_z(t)$ in the cavity shows that the solution decays more rapidly than that of the previous cycle in the coasting phase, indicating a lower quality factor. Values in the energy

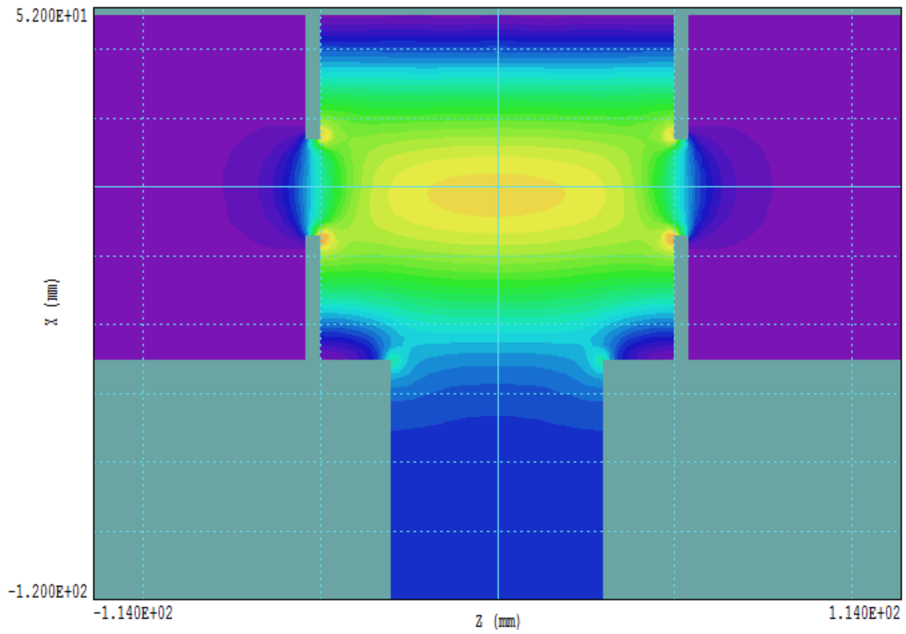

Figure 12: Waveguide driven by a resonant cavity.

summary of the **Aether** listing file give $Q = 147.9$. Figure 12 shows the variation of $|\mathbf{E}|$ in the plane $y = 0.0$ mm. The $\mathtt{TM}_{010}$ mode fields are slightly distorted and the electric field amplitude in the waveguide is almost constant, indicating a pure traveling wave. Using the same analysis techniques as in the previous example, I found that the voltage stepdown ratio was 1.0/7.3.

The inverse process, a transmission line or waveguide driving a resonant cavity, is more subtle. It is tempting to think that we could model the process by inverting the solution – locate the drive current in the input line to make a traveling wave moving toward the cavity and add conductivity to the in-cavity load to achieve the same cavity Q factor. The problem is that the actual physical state is not an inversion. When the cavity resonance reaches equilibrium, there is a traveling wave of equal amplitude moving away from the cavity, driving the impedance of the line. A cavity load adds to this energy loss. The result is that the transmission line or waveguide supports waves moving in both directions, giving a standing wave pattern. As a test, I performed an **Aether** calculation for the waveguide example with an appropriate source located in the line (following Chapter 9 of the **Aether Tutorial Manual**). With no cavity loading, power loss resulted solely from the backward moving wave. Equibrium was achieved after about 150 cycles. The line carried almost a pure standing wave. Both the Q value and the voltage stepdown ratio were the same as the previous extraction solution. The net effect was simply to change the location of the drive current.