Scaling in numerical calculations: the strip transmission line

Stanley Humphries, Ph.D.

Field Precision LLC
E mail: techinfo@fieldp.com
Internet: https://www.fieldp.com

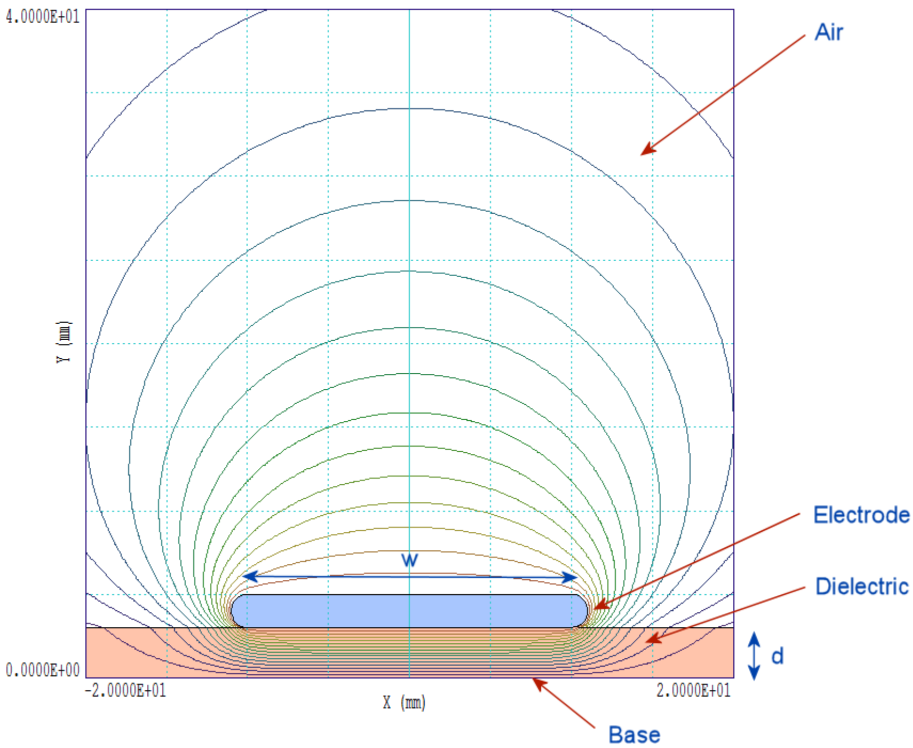

Figure 1: Geometry of the strip transmission line with dimensions for the example calculation ($d = 3.0$ mm and $w = 10.0$ mm).

In a previous tutorial[1], I discussed some relationships between analytic and numerical calculations in physics and how analytic results are useful to confirm numerical work and to put it in perspective. In this tutorial, I will use the another familiar example of the strip transmission line used in high-frequency integrated circuits to show another aspect of the relationship between analytic and numerical approaches, the use of scaling relationships to organize and to generalize numerical results.

Figure 1 shows the geometry, a cross-section of a long assembly. An electrode rests on a dielectric sheet that in turn rests of a grounded base plane. The dielectric sheet has thickness d and the electrode has width w. We assume that there are no nearby objects, so there is a large space with $\epsilon_r = 1.0$ and $\mu_r = 1.0$ above. The dimensions in Fig. 1 are in millimeters. Note that the size of the assembly is considerably larger than might be encountered in an integrated circuit. This is not a problem. We'll express the results in a form so that the ratio w/d is important, not the absolute dimensions.

[1] *Analytic versus numerical approaches: the two-wire transmission line*, `https://www.fieldp.com/tutorials/TwoWire.pdf`

A temptation in numerical calculations is to represent the target system exactly, a literal simulation. Instead, we'll organize the work to represent the general class of strip transmission lines, giving the numerical calculation some of the predictive capability of an analytic solution. In preparation, consider the theory of the simple strip line presented in most introductory texts on electromagnetism. The assumption is that $w \gg d$ so that field energy outside the gap between the electrode and ground can be neglected. Assuming that $\mu = \mu_0$ in the dielectric, the following relations hold for the inductance per length in z, the capacitance per length and the characteristic impedance:

$$C_\infty = \epsilon_r \epsilon_0 \, \frac{w}{d}, \qquad \text{(F/m)} \tag{1}$$

$$L_\infty = \mu_0 \, \frac{d}{w}, \qquad \text{(H/m)} \tag{2}$$

$$Z_\infty = \frac{d}{w} \sqrt{\frac{\mu_0}{\epsilon_r \epsilon_0}}. \qquad \Omega \tag{3}$$

The purpose of the numerical calculation is to investigate more practical cases where w is comparable to d. The strategy is that rather than dealing with absolute numbers of individual geometries, we will organize results to make comparisons with the infinite-width predictions. This approach has two advantages:

- Confirmation that the numbers are in the right ballpark so there is probably not a fundamental mistake in the setup.
- Generalizing the results so that the apply to strip lines of any dimension.

The parameters for the calculation are $d = 3.0$ mm and $\epsilon_r = 2.8$ (in the range of many plastics and ceramics). I made solutions for several values of w (5.0, 10.0, 20.0, 30.0, 40.0 and 50.0 mm) with the idea that intermediate results could be determined by interpolation. The corresponding range of w/d is 1.67 to 16.67. The electrostatic and magnetostatic calculations are performed in a box 100 mm on a side to minimize boundary effects. We will check the validity of this assumption later. Figure 1 shows a zoomed view of the electrode region for the **EStat** calculation of the capacitance. Region 1 is an air volume ($\epsilon_r = 1.0$) that fills the solution box. Region 2 is a dielectric slab of thickness 3.0 mm that over-writes the bottom section of the solution box. Region 3 is an electrode with a fixed potential $V_0 = 1.0$ V. The electrode has thickness 2.0 mm with rounded edges to avoid undefined fields at sharp corners that would compromise the accuracy of the energy integrals. Finally, Region 4 sets nodes on the boundary of the solution volume to the fixed potential condition $V = 0.0$ V. The region represents the bottom plane and reflects the assumption that structures remote from the line are near ground potential.

The run uses six **Mesh** input files to represent electrodes of different widths. Preparation is relatively straightforward using the **Mesh** drawing editor. After creating one assembly, the mesh is loaded into the editor. The top and bottom lines of the Region 3 are deleted, the end arcs are selected and moved, the top and bottom lines redrawn and the new mesh saved with a name descriptive of the width. Similarly, six **EStat** input files can be quickly prepared by changing the **Mesh** input file reference of the base script and saving the modified files with different names. The full set of six calculations can be performed in batch mode in less that half a minute. A single button click in the postprocessor determines the global field energy per length (U), and the capacitance follows from

$$C = 2\,\frac{U}{V_0^2} = 2U. \tag{4}$$

Given the set of capacitance values C_n, the question is what is the best way to display the data? Notice in Eqs. 1, 2 and 3 that all quantities involve the ratio w/d. This is a good choice for the independent variable. For the dependent variable, we shall use C_n/C_∞. As a check, if we did everything correctly the value should approach 1.0 at large values of w/d. Furthermore, this choice clearly shows the relative effects of narrow electrodes compared to the infinite electrodes of simple model. The curve with green markers in Fig. 2 shows the results. As expected, the curve does tend toward the value 1.0 at large w although there is a significant discrepancy even at $w = 50.0$ mm. The capacitance is much larger than that of the simple model at small w/d, reflecting the fact that the fringe fields spread out over a width larger than w. There are two main advantages to the choice of display:

- Interpolations between the six calculation points give good estimates of capacitance per length for intermediate values of w/d.
- The results can be applied to strip lines with similar electrode shapes of any size.

Before we get carried away with generality, we need to recognize that the values of ϵ_r hidden in the scaling may have an effect. The discrepancies from the infinite values results from fringing fields, and their distribution is affected by the dielectric constant of the sheet. A high value tends to squeeze more of the electric field distribution into the electrode gap, reducing the difference from the infinite plate model at low w, as shown in Fig. 3. Repeating the

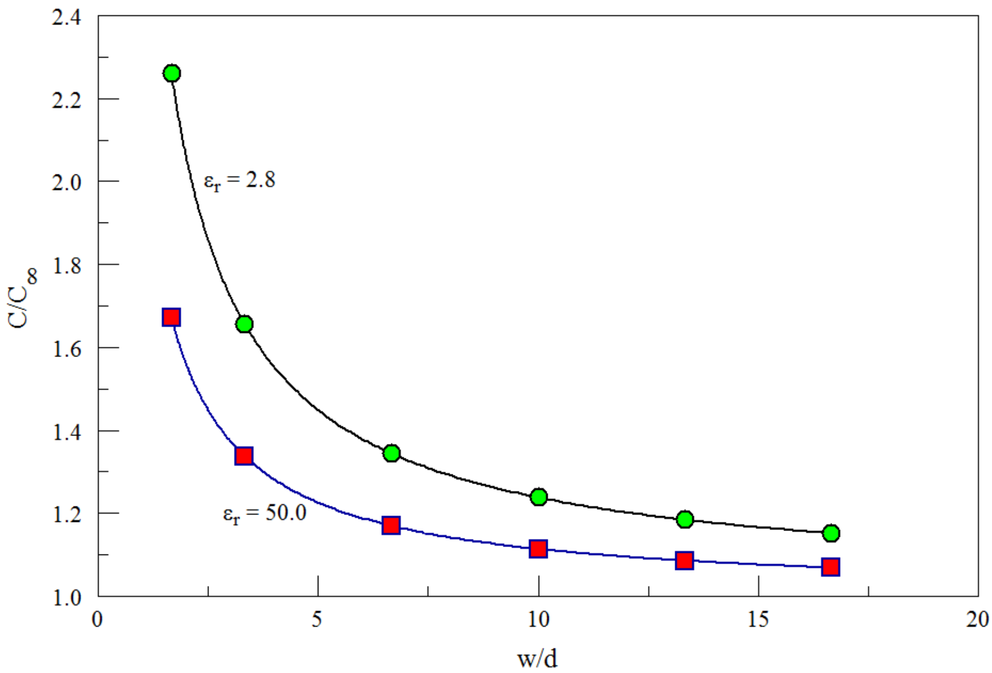

Figure 2: Ratio of the capacitance per length of a a strip transmission line to the value for an infinite width line as a function of w/d for two values of relative dielectric constant. Calculated values with fitted connecting curves.

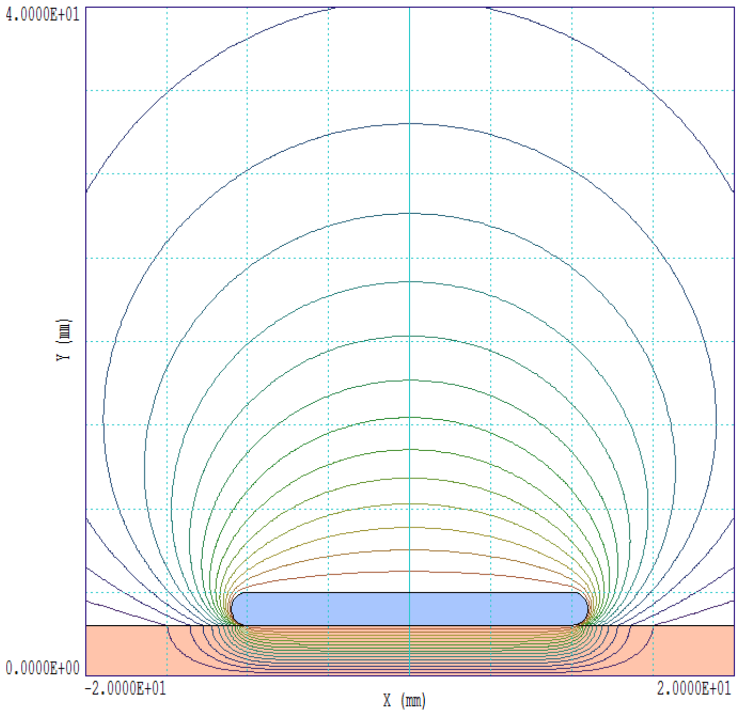

Figure 3: Equipotential lines for $w = 10.0$ mm with a high dielectric constant, $\epsilon_r = 50$.

calculations with $\epsilon_r = 50.0$ gives the curve with red markers in Fig. 2. My general conclusion is that the green curve of Fig. 2 gives good initial design guidance for common dielectrics in the range $\epsilon_r = 2.0 \rightarrow 4.0$. Initial estimates could be followed with numerical calculations with specific electrode shapes and values of ϵ_r.

We'll advance to the calculation of inductance per unit length. Because it is unlikely to encounter insulating materials with $\mu \neq \mu_0$, these results have broad generality. To set up the **PerMag** calculation, I removed the dielectric and added a new drive region inside the electrode (Fig. 4). The goal is model high-frequency waves where the electrode excludes the magnetic field and current flows in a thin layer on the surface. The previous tutorial discussed an easily-implemented approach. The electrode is assigned a low value of magnetic permeability ($\mu_r \ll 1.0$) so that it acts as a field excluder. For a drive current of $I_0 = 1.0$ A, the inductance is related to the field energy in the air region by

$$L = 2\,\frac{U}{I_0^2} = 2U. \tag{5}$$

The main considerations for a good run are to provide sufficient elements to

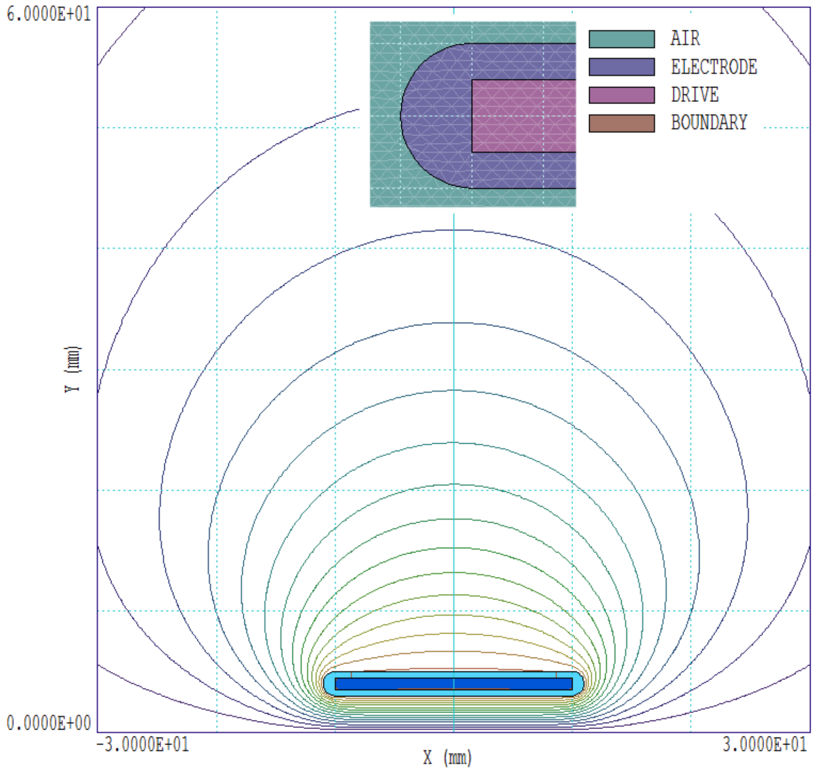

Figure 4: Geometry for the inductance calculation with lines of magnetic induction **B**. The electrode of width w = 10.0 mm carries 1.0 A. Dark blue: current source. Light blue: field shaper with $\mu_r = 0.001$.

resolve the material properties (Fig. 4) and to allow sufficient run time for complete convergence.

Figure 5 shows the normalized results. The inductance is lower at smaller w because the field lines cover a much larger volume than that of the gap. Even at large w, the results differ significantly from the infinite w approximation. For example, at w = 40.0 mm, the inductance is only 75% of the simple theoretical prediction. We can check out the nature of the discrepancy by doing a run where the air is region divided in two: the gap between the electrode and ground plane and the remainder of the volume. The **PerMag** code can then perform an energy integral organized by region. The result is that the energy associated with fringing flux and return flux constitutes 25.4% of the total energy. I also ran check to show that the difference was not a boundary effect by doubling the area of the solution volume. There was a negligible difference in the energy calculation.

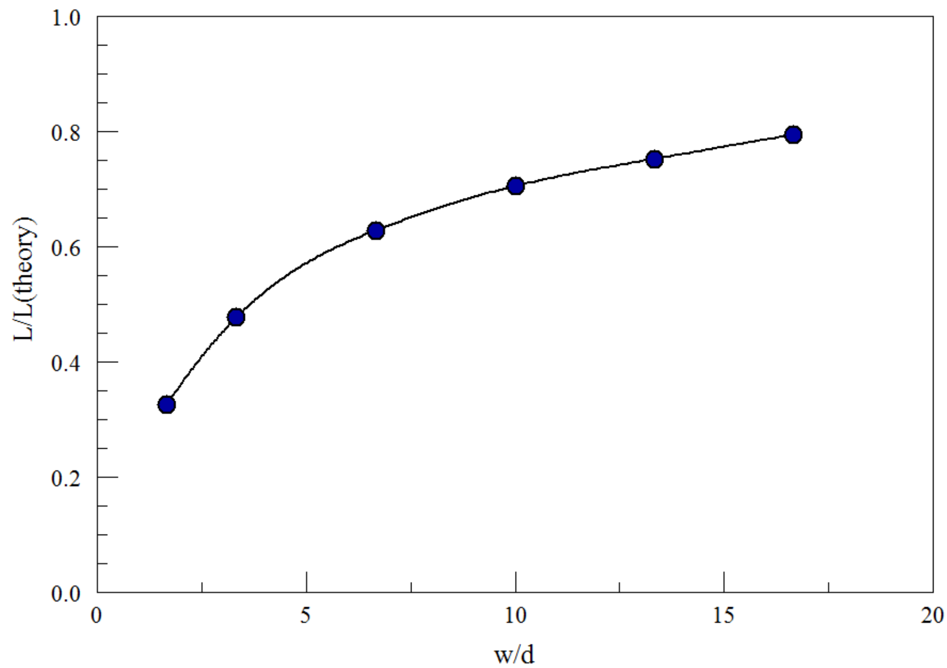

Figure 5: Normalized inductance per length, L/L_∞, as a function of w/d, universal curve for strip transmission lines.

To conclude, Fig. 6 shows a normalized plot of the transmission line characteristic impedance at $\epsilon_r = 2.8$. In summary, this report emphasized some useful techniques to make numerical studies more effective:

- Make comparisons with analytic results when possible to check whether solutions are reasonable.
- Use scaling to normalize results – it is easier to see trends in data with values near unity rather than very small or large absolute values.
- Use the code as an exploratory tool to confirm results.

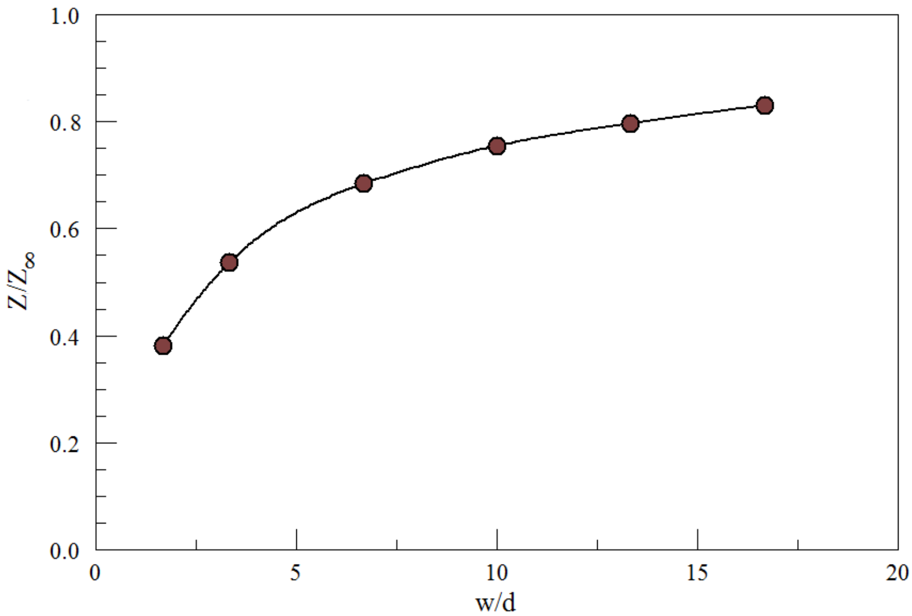

Figure 6: Characteristic impedance of a strip transmission line as a function of normalized width (w/d) with a insulator dielectric constant of $\epsilon_r = 2.8$.