Current flow through a shaped foil resistor

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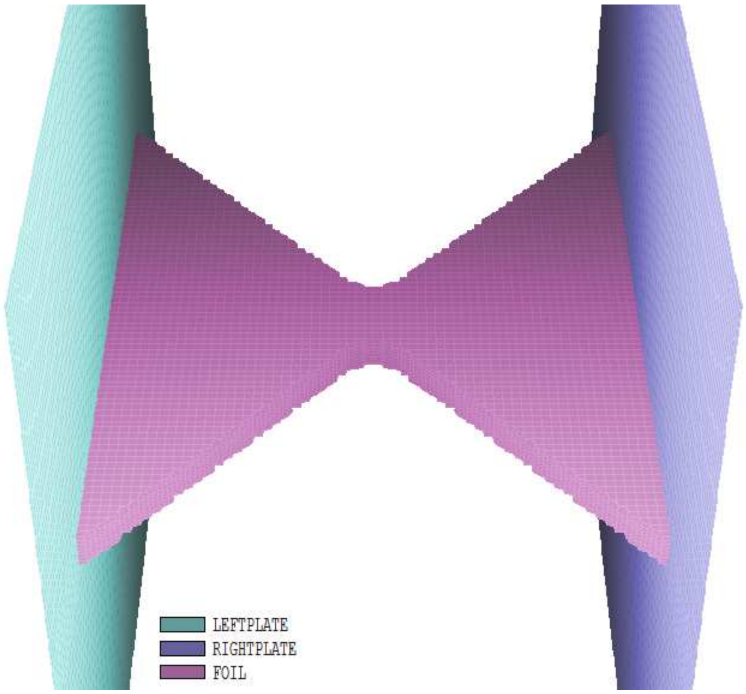

Figure 1: Geometry of the foil resistor, meshed for a 3D calculation. Note that the thickness is 2.0 mm, a hundred times that of the 20 μm foil thickness in the experiment.

I had a recent inquiry about modeling current flow though a shaped foil conductor in support of experiments on shock initiation. The initial goal was to determine the distribution of current density and power distribution before significant foil heating. Figure 1 shows the geometry. The foil spans a 30 mm air gap between high voltage electrodes. The foil has 40 mm width at the connection point and width 5.3 mm at the neck. The foil thickness is only 20 um. At first glance, this appears to be a challenging problem for a 3D finite-element code because of the large difference in scale sizes. In reality, the solution can be resolved to a simple two-dimensional electrostatic conductive calculation because of the following factors:

- The current rise-time (limited by the pulser inductance) is long enough so that the effective skin depth is greater than the foil thickness. The implication is that inductively driven currents are small compared to the real current through the foil.
- In addition, the conductive current is much larger than displacement currents, so an electrostatic solution is sufficient.
- The field distribution in the air region surrounding the foil does not effect the current distribution in the foil.

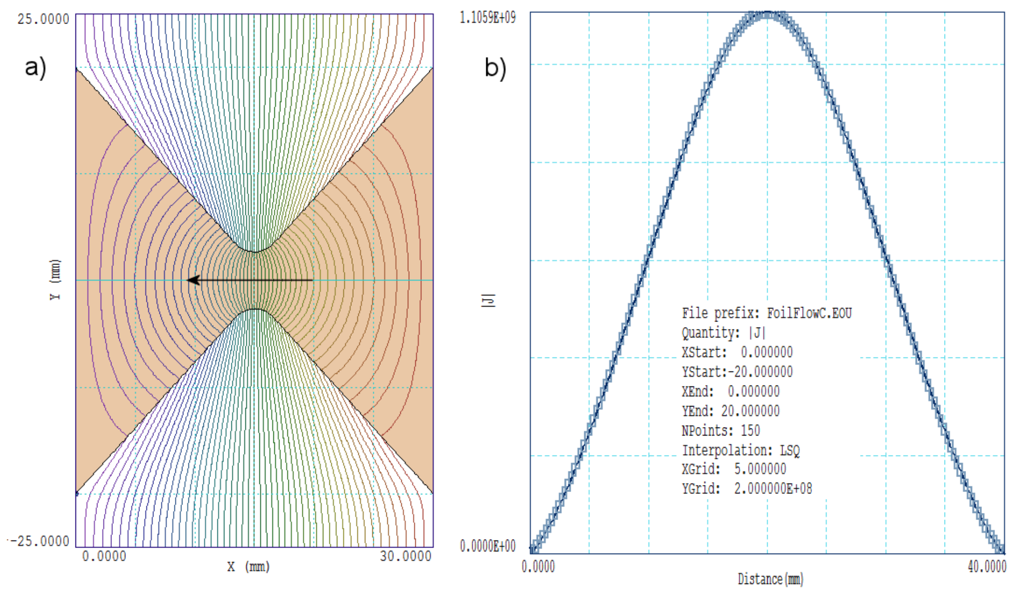

Figure 2: Two-dimensional **EStat** solution. *a*) Cross-section geometry with equipotential lines. Current flows normal to the lines. *b*) Scan of current density over the foil at the left-hand boundary.

The implication is that there is no potential difference across the narrow dimension of the foil. Therefore, the flow of conductive current would be unaffected if we added an identical adjacent foil. The extra foil would change external fields, but would not affect the current distribution in the first foil. By this reasoning, we could add an infinite set of adjacent foils. The geometry of the resulting system is planar – the foil becomes an extrusion with infinite length in z and geometric variations in x and y. A 2D electrostatic code could then applied. I will first discuss the 2D solution, and then review a 3D calculation to confirm the validity of assumptions.

The setup in the **EStat**[1] code takes just a few minutes. Figure 2 shows the geometry and calculated results. In building the conformal triangular mesh, Region 1 fills the solution volume, Region 2 is a long bar with a cross section in the shape of the foil that over-writes elements of Region 1. Region 3 is the set of nodes on the left boundary and Region 4 comprises the nodes of the right boundary. The control script for the **EStat** solution has the following content:

[1]https://www.fieldp.com/estat.html

```
Mesh = FoilFlow
Geometry = Rect
DUnit =   1.0000E+03
* Region 1: AIR
Sigma(1) =   1.0000E+00
* Region 2: FOIL
Sigma(2) =   5.7970E+07
* Region 3: LEFTBOUND
Potential(3) =  -5.0000E-01
* Region 4: RIGHTBOUND
Potential(4) =   5.0000E-01
EndFile
```

The appearance of `Sigma` commands signals that the solution is conductive. There is a voltage difference of 1.0 V between the boundaries on the left-hand and right-hand sides. The top and bottom boundaries assume the default Neumann condition (where equipotential lines are normal). The foil has the conductivity of copper and air is assigned a much lower conductivity. The non-zero value is required for convergence. Current density flows normal to the equipotential lines of Fig. 2a. With the large difference in conductivity, the current flow inside the foil is unaffected by the field distribution in air. The potential distribution along the foil periphery provides a boundary condition for the Laplace equation solution in the surrounding air volume.

Figure 3 shows a plot of the resistive power density, p $= j^2/\sigma$, showing the high concentration at the neck. In the analysis mode, **EStat** provides several options to determine the resistance of the structure. For example, Fig. 2b shows a scan of the linear current density $|J|$ along the left boundary. Several operations are performed in response to this analysis script:

```
INPUT FoilFlow.EOU
OUTPUT FoilFlowAnalysis.DAT
VOLUMEINT 2
SURFACEINT 4 -2
LINEINT 14.99 -20.0  14.99 20.0
ENDFILE
```

The volume integral gives the result $p = 2.3717\times10^7$ W/m (power dissipation per meter length in z). Multiplying by the foil thickness, the total power dissipation in a foil of thickness 20 μm is P = 473.34 W. With the 1.0 V applied voltage, the resistance is $R = 2.113$ mΩ implying a total current I = 473.34 A. The surface integral of current flow from the right-hand boundary into the foil (the integral of Fig. 2b) is $J = 2.3717\times10^7$ A/m. Multiplying by the foil thickness, this method predicts a total current $I = 474.34$ A. Finally, the line integral result yields a total current 470.88 A. Results of the three methods are consistent to within $\pm0.4\%$.

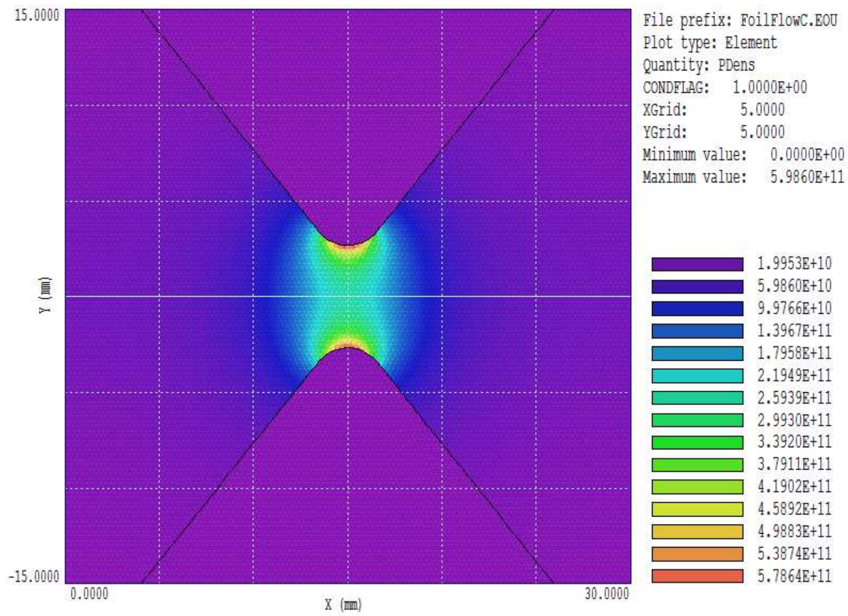

Figure 3: Resistive power deposition j^2/σ in W/m^3.

To confirm the assumptions of the analysis, we can do a 3D solution using the mesh of Fig. 1. Because the thickness of the foil should not affect the current distribution over the cross section, we will facilitate the finite-element analysis by using a thickness of 2.0 mm rather than 20 μm (differing by a factor of 100.0). The run-time of the **HiPhi**[2] solution with 857,285 elements is 51 seconds. Figure 4 shows results. The top equipotential plot shows a cut across the direction of current propagation 5.0 mm from one of the drive electrodes. The plot confirms that the potential is uniform across the thickness of plate. The lines in the air region are quite different from those of a foil with infinite length in z. Nonetheless, the relative distribution of current density inside the conductor is the same. For comparison, Fig. 4b shows the power density distribution a slice normal to z. The results are almost identical to the 2D results of Fig. 3, the small difference resulting from different size and shape of the computational elements.

The **PhiView** analysis script has the content:

```
INPUT FoilFlow3D.HOU
OUTPUT FoilFlowAnalysis.DAT
VOLUMEINT 4
SURFACEINT 3 -4
ENDFILE
```

[2]https://www.fieldp.com/hiphi.html

The integral of power over the foil volume is 4.7186×10^4 W. Dividing by a factor of 100.0 gives almost a result almost exactly equal to the 2D calculation scaled to the 20 μm foil. Similarly, the surface integral over the contract area between the electrode and foil is 4.7249×10^4 A, again 100 times the result for the thin foil.

Finally, it's always a good idea to check that numerical results are in the right range with an analytic solution. Figure 5 shows the geometry. The foil cross section is approximated with a butterfly shape. The dimensions are $W_1 = 4.0 \times 10^{-2}$ m, $W_2 = 5.0 \times 10^{-3}$ m, $L = D/2 = 1.5 \times 10^{-2}$ m and the thickness is $\delta = 2.0 \times 10^{-5}$ m. The idea is to divide the foil into small resistors of length dz and add them in series. The resistance of one element is

$$dR = \frac{\rho dz}{\delta(W_1 - z(W_1 - W_2)/L}. \tag{1}$$

where ρ is the volume resistivity of copper, 1.725×10^{-8} Ω-m. Defining $Z = z/L$ and taking the integral from $z = 0$ to $z = L$, the total resistance of both butterfly sections is approximately

$$R = \frac{\rho D}{\delta W_1} \int_0^1 dZ \; \frac{1}{1 - Z(1 - W_2/W_1)}. \tag{2}$$

The initial factor in Eq. 2 is the resistance of a uniform foil of width W_1, length D and thickness δ. The value is 6.46×10^{-4} Ω. The integral can be viewed as a correction factor to account for the foil shape. Applying a table of integrals, the value is

$$\int_0^1 dZ \; \frac{1}{1 - Z(1 - W_2/W_1)} = -\frac{1}{1 - W_2/W_1} \; \ln(W_2/W_1) = 2.377. \tag{3}$$

The predicted resistance is $R = 1.53$ mΩ compared to the code result of 2.12 mΩ. We can conclude that the numerical results are reasonable. The difference is not surprising. The underlying assumption of the simple model of Fig. 5 is that current is uniformly distributed over the height of the slices. Figure 2b shows that the condition does not hold in a full solution, so we expect that the analytic estimate will be low.

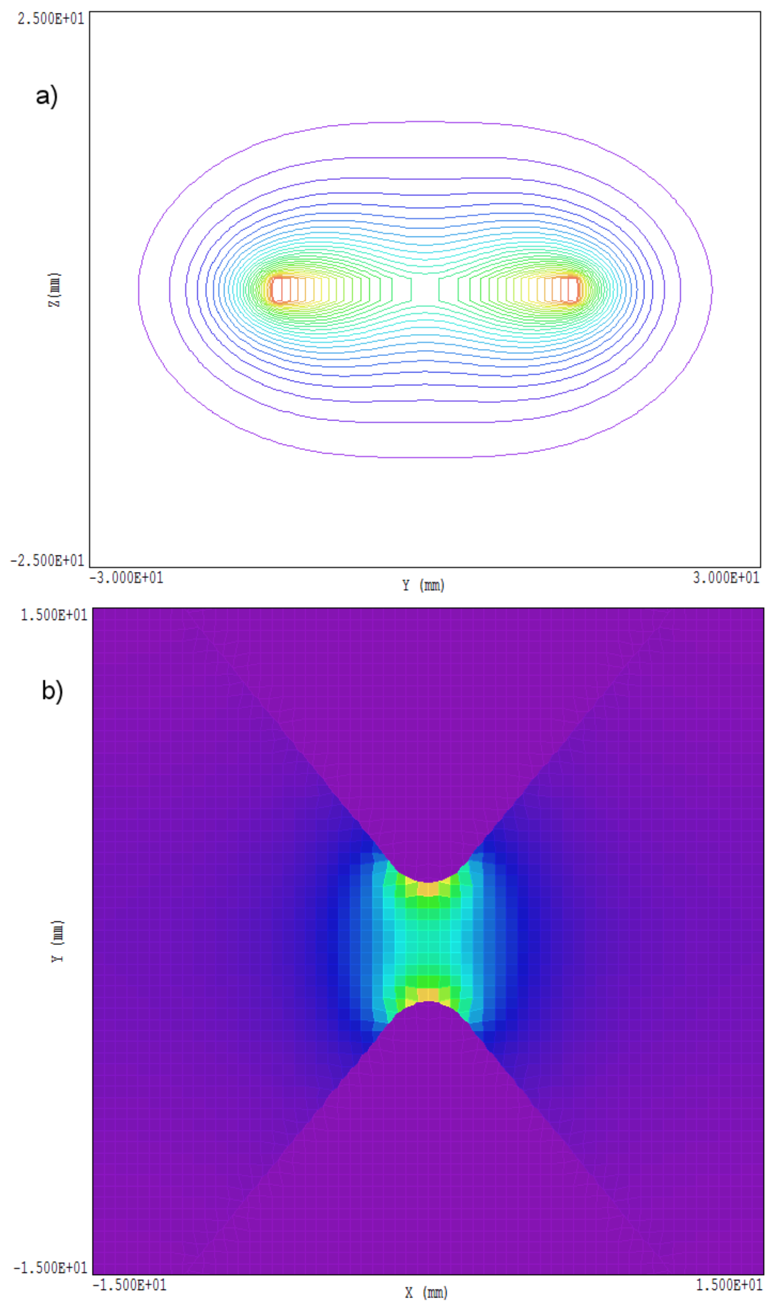

Figure 4: **HiPhi** solution using the mesh of Fig. 1. a) Equipotential lines, slice normal to x to the direction of current flow through the foil, 5 mm from the electrode. b) Slice normal to z, power density profile using the same scaling as Fig. 3.

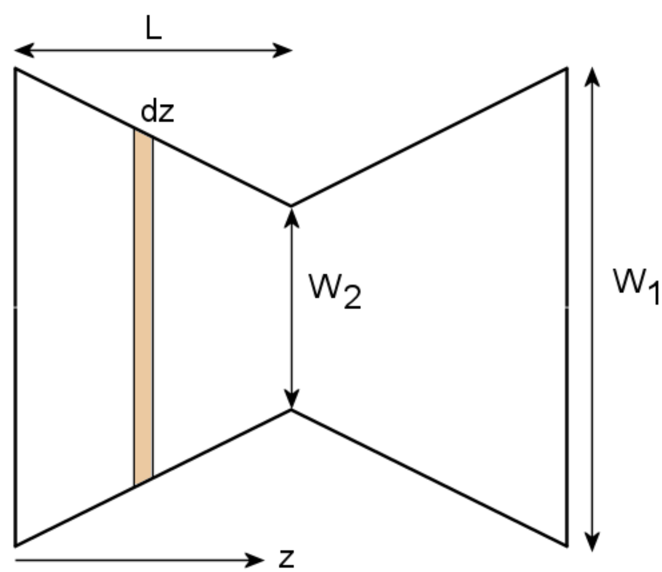

Figure 5: Geometry for estimating the resistance of a shaped foil.